SUPPLEMENT

THUNDERSTORMS DO NOT GET BUTTERFLIES

BY DALE R. DURRAN AND JONATHAN A. WEYN

This document is a supplement to "Thunderstorms Do Not Get Butterflies," by Dale R. Durran and Jonathan A. Weyn (*Bull. Amer. Meteor. Soc.*, **97,** 237–244) • ©2016 American Meteorological Society • *Corresponding author:* Dale Durran, Department of Atmospheric Sciences, University of Washington, Box 351640, Seattle, WA 98195-1640 • E-mail: drdee@uw.edu • DOI:10.1175/BAMS-D-15-00070.2

IMPLEMENTATION OF THE LORENZ MODEL. The mathematical formulation of the Lorenz turbulence model follows that described previously (Lorenz 1969; Rotunno and Snyder 2008). The curves in Fig. 1 in the main text are produced using Lorenz's original dynamical formulation based on linearized barotropic vorticity dynamics with the saturation background kinetic energy spectrum given by Eq. (52) in Lorenz (1969). In contrast to previous studies (Lorenz 1969; Rotunno and Snyder 2008), a smooth approach to nonlinear saturation is enforced by solving the system of ordinary differential equations (Durran and Gingrich 2014):

$$\frac{dY_k}{dt} = \sum_{l=1}^{n} C_{k,l} Z_l, \quad \frac{dZ_k}{dt} = \left(\frac{X_k - Z_k}{X_k} \right) Y_k.$$

Here n is the total number of spectral bands, and Z_k is the ensemble mean of the kinetic energy of the perturbation velocities about the ensemble mean, integrated with respect to $\ln(k)$ over the spectral band at two-dimensional horizontal wavenumber k (the wavenumber is 2π divided by the wavelength). The background kinetic energy spectrum X_k is again integrated with respect to $\ln(k)$. A dummy variable Y_k is used to write a second-order system of differential equations as a larger first-order system. Adjacent wavenumbers in our truncation differ by a factor of $\sqrt{2}$, which is twice the spectral resolution used by Lorenz (1969) and Rotunno and Snyder (2008); we retain 40 wavenumbers. Following Lorenz (1969), the shortest retained wavelength is 38 m and the longest length scale is $L = 40{,}000$ km; the total integrated background kinetic energy is set to $E = 148$ m^2 s^{-2}, and the dimensional scale for time is $LE^{-1/2}$. The MATLAB code used to run the Lorenz model is available from the authors.

THE SQUALL-LINE SIMULATIONS. The numerical model solves the nonlinear, nonhydrostatic, compressible equations of motion together with a warm-rain microphysics parameterization (Durran and Klemp 1983); Coriolis forces are neglected. The numerical domain is periodic along both horizontal coordinates (thereby facilitating the spectral analysis) and spans a distance of 512 km in both the east–west coordinate x and the north–south coordinate y. The upper boundary permits gravity wave radiation (Klemp and Durran 1983; Bougeault 1983). The grid spacing is $\Delta x = \Delta y = 1$ km with Δz gradually increasing from 200 m near the surface to 500 m above 5 km. The model is integrated using a two-time-step algorithm in which the large time step is 3 s and the acoustic time step is 1.5 s.

The idealized atmospheric environment in which the squall line grows is similar to that in Weisman and Klemp (1982). A unidirectional horizontally uniform background wind profile is specified in which winds from the west increase from 0 m s^{-1} at the surface to 20 m s^{-1} at a height of 5 km and remain 20 m s^{-1} at higher levels. The background north–south wind component is zero. The vertical profile of potential temperature and relatively humidity (RH) for the horizontally homogeneous initial state is taken from Eqs. (1) and (2) in Weisman and Klemp (1982), with a constant mixing ratio of 14 g kg^{-1} between the surface and 1 km, except that the RH is capped at 90% for $1 \le z \le 1.4$ km and at 75% at any level above 1.4 km where Eq. (2) would yield a higher value (Wandishin et al. 2008).

Three localized warm bubbles with positive temperature perturbations 2 K warmer than their environment trigger the initial updrafts, which owing to the wind shear ultimately evolve into a line of thunderstorms (Skamarock et al. 1994). These bubbles are spheroidal with 2-K perturbations in their center, a 10-km horizontal radius, and a 1.4-km vertical radius. Their centers are 1.4 km above the surface, and their (x,y) coordinates are (100, 250), (150, 300), and (125, 175) km. Variations in the position and strength of the squall line within each 25-member ensemble are produced by small-amplitude background perturbations in the surface temperature field that decay exponentially with height over an e-folding scale of 1 km. In terms of potential temperature θ′, these perturbations have the form

$$\theta' = ae^{-z}\sin\left[2\pi\left(\frac{x}{L}-\phi_x\right)\right]\sin\left[2\pi\left(\frac{y}{L}-\phi_y\right)\right].$$

Here the phase of the perturbation in each ensemble member is determined by the random numbers (φ_x,φ_y) in the interval [0,1]; L is either 8 km in the small-scale ensemble or 128 km in the large-scale ensemble; and x, y, and z are distances in kilometers along each coordinate. The amplitude a is set to 0.5 K for the 8-km-scale perturbations and is reduced by a factor of one-quarter to 0.125 K for the 128-km perturbations.[1] The synthetic radar data displayed in Fig. 2 in the main text are computed from the simulated rain fields (Koch et al. 2005).

THE KE SPECTRUM. The ensemble-mean kinetic energy (KE) spectrum for the squall-line simulations was computed as follows. At a given vertical level, let $u_{i,j,m}$ and $v_{i,j,m}$ denote the zonal and meridional velocities, respectively, at horizontal mesh point (i,j) for ensemble member m. The discrete Fourier transform was applied along the x coordinate to each $u_{j,m}$ and $v_{j,m}$. Denoting the transform of a function φ by $\hat{f}$, and the complex conjugate by φ^*, the kinetic energy spectral density for each j and m was computed as

$$\widehat{\mathrm{KE}}_{j,m}(\tilde{k}) = \frac{\Delta x}{2N_x}\left[\hat{u}_{j,m}(\tilde{k})\,\hat{u}^*_{j,m}(\tilde{k}) + \hat{v}_{j,m}(\tilde{k})\,\hat{v}^*_{j,m}(\tilde{k})\right].$$

Here, $\tilde{k}$ is the (one dimensional) wavenumber along the x coordinate ($\tilde{k} = 2\pi/\lambda_x$, where λ_x is the wavelength parallel to the x axis), and $N_x = 512$ is the total number of grid points. Then, $\widehat{\mathrm{KE}}_{j,m}(\tilde{k})$ was averaged over both j and m to give the ensemble- and meridionally-averaged one-dimensional total KE spectrum. The perturbation kinetic energy spectral density KE′ is calculated in the same manner as KE, except that $u_{i,j,m}$ and $v_{i,j,m}$ are replaced with $u'_{i,j,m} = u_{i,j,m} - \langle u_{i,j}\rangle$ and $v'_{i,j,m} = v_{i,j,m} - \langle v_{i,j}\rangle$, respectively, where $\langle\varphi\rangle$ indicates the average of φ over all 25 ensemble members. Only those wavelengths greater than $7\Delta x$ are shown in Fig. 3 in the main text; $7\Delta x$ is the (slightly arbitrary) cutoff scale beyond which Skamarock (2004) found numerical dissipation excessively reduced KE in mesoscale numerical models. The normalized perturbation kinetic energy plotted in Fig. 5 in the main text is computed as

$$\mathrm{KE}'_n(\tilde{k}) = Ak^{-5/3}\frac{\mathrm{KE}'(\tilde{k})}{\mathrm{KE}(\tilde{k})}, \qquad \text{(ES1)}$$

where A is a constant mapping of the result to the observed mesoscale background KE spectrum.

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