Relationships between optical depth, liquid water path, droplet concentration and effective radius in an adiabatic layer cloud

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For a layer cloud in which the liquid water content $q_L(z)$ increases linearly with height z above cloud base, we can write

$$q_L(z) = f_{ad}\Gamma_{ad}z \tag{1}$$

where $\Gamma_{ad}(T, p)$ is the adiabatic rate of increase of liquid water content with respect to height. This is a fairly weak function of temperature T and pressure p, and so can be assumed to be approximately constant with height in shallow layer clouds. The constant f_{ad} is in the range $0 < f_{ad} < 1$ and is the degree of adiabaticity. The liquid water path LWP is defined as

$$LWP = \int_{z=0}^{h} q_L dz = \frac{1}{2} f_{ad}\Gamma_{ad}h^2 \tag{2}$$

where h is the cloud thickness. The effective radius r_e is defined as the ratio of the third to second moments of the droplet size distribution. The mean volume radius r_{vol} is defined through the relationship between droplet concentration N_d and q_L:

$$r_{vol} = \left(\frac{3q_L}{4\pi\rho_w N_d}\right)^{1/3} \tag{3}$$

where ρ_w is the density of liquid water. The effective radius and volume radius can be related by a constant k, where:

$$k = \left(\frac{r_{vol}}{r_e}\right)^3 \tag{4}$$

Optical thickness is related to liquid water content and effective radius via

$$\tau = \int_{z=0}^{h} \frac{3q_L}{2\rho_w r_e} dz \tag{5}$$

where we make the assumption that $Q_{ext} = 2$. Using [5], [4], [3] and [2] the optical thickness can be expressed as a function of cloud droplet concentration and cloud thickness

$$\tau = A\Gamma_{eff}^{2/3} N_{eff}^{1/3} h^{5/3} \tag{6}$$

where $\Gamma_{eff} = f_{ad}\Gamma_{ad}$, $A = \left(243\pi/250\rho_w^2\right)^{1/3}$=0.0145 and we define an effective concentration as $N_{eff} = kN_d$. Similarly, the cloud top effective radius $r_e(h)$ can be expressed as a function of cloud thickness and droplet concentration:

$$r_e(h) = B\Gamma_{eff}^{1/3} N_{eff}^{-1/3} h^{1/3} \tag{7}$$

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with $B = (3/4\pi\rho_w)^{1/3}$=0.0620. Combining [6] and [7] by multiplication leads to an expression relating τ, LWP and $r_e(h)$

$$LWP = \frac{5}{9}\rho_w \tau r_e(h) \tag{8}$$

Equation [8] can be used to derive liquid water path using satellite retrievals of optical depth and cloud top effective radius.

A measure of droplet concentration can be retrieved from satellite measurements of $r_e(h)$ and τ by eliminating h from [6] and [7]:

$$N_{eff} = \frac{\sqrt{10}}{4\pi\rho_w^{1/2}}\Gamma_{eff}^{1/2}\frac{\tau^{1/2}}{r_e(h)^{5/2}} \tag{9}$$

or in terms of LWP and $r_e(h)$:

$$N_{eff} = \sqrt{2}B^3\Gamma_{eff}^{1/2}\frac{LWP^{1/2}}{r_e(h)^3} \tag{10}$$

Because mean values of k for most marine clouds are approximately 0.8, the value of N_{eff} will be 20-30% lower than N_d in most clouds. Observations suggest that within any cloud the value of k can be quite variable. Retrieval of k values is not yet possible.

Values of Γ_{ad} are shown in Fig. 1 as function of T and p. For most clouds lower than 800 hPa, the value of Γ_{ad} varies by only around 20%. The value of f_{ad} is often quite close to unity in nonprecipitating stratocumulus, although there are few definitive measurements.

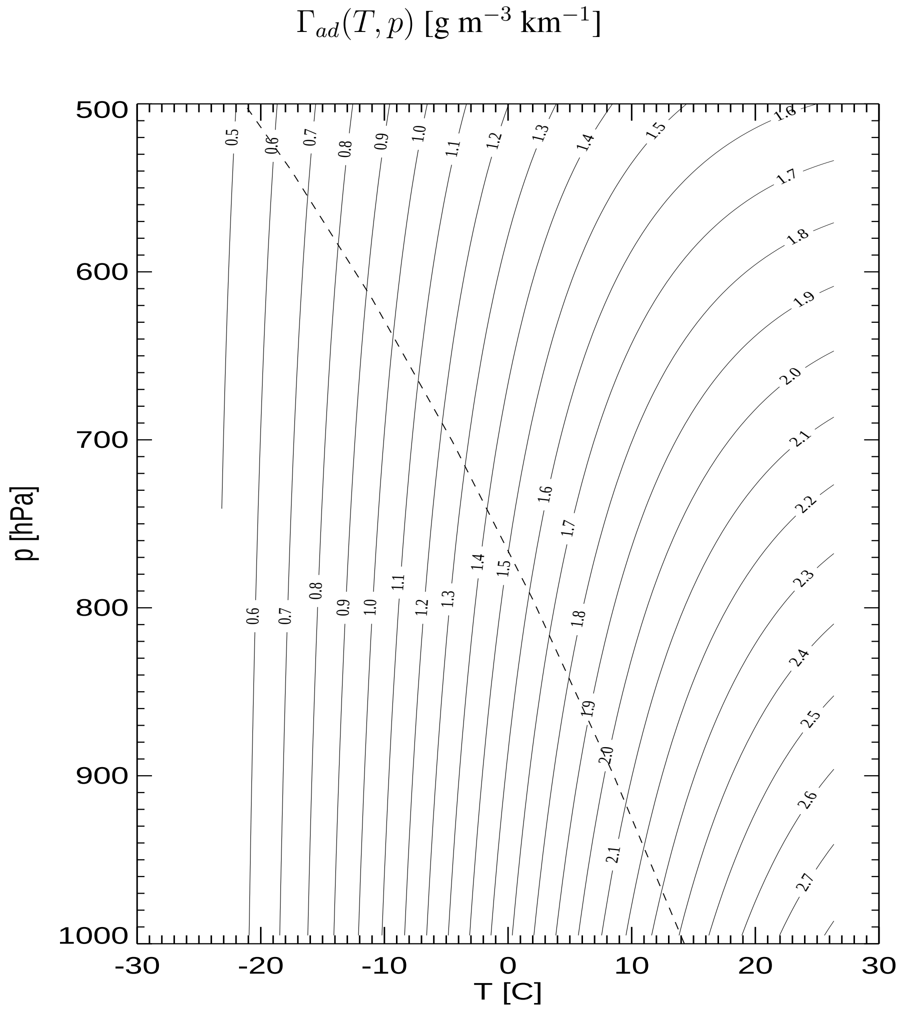

Figure 1: Values of Γ_{ad} as a function of temperature and pressure. The ICAO standard atmosphere is shown by the dashed line.