

WEEK 5/6:
TEMPERATURE AND HUMIDITY in the BOUNDARY-LAYER
Homework assignment

Problems:

1. Explain why the saturated adiabatic lapse rate is less than the dry adiabatic lapse rate. Under what conditions would you expect the largest and smallest deviations of the saturated lapse rate from the dry one? What does this imply for the climatological distribution of Foehn winds. [2pts]

2. Sketch realistic profiles of virtual potential temperature θ_v and specific humidity for a surface-driven convective boundary layer at noon. On the θ_v profile indicate the regions where the profile is stable and unstable according to both local and non-local stability categorizations. [2pts]

3. (a) Calculate the increase in temperature that will be experienced in 3 hours by a mixed layer of depth 1000 m given a surface sensible heat flux of 300 W m^{-2} . You can assume that no energy is transferred above the top of the mixed layer.

(b) Assuming that the mean relative humidity at the surface is 80%, estimate the specific humidity given a mean initial surface temperature of 15°C and a surface pressure of 1013 hPa. You will need to locate an expression for the saturation vapor pressure as a function of temperature to do this.

(c) Calculate the mean surface virtual temperature.

[3pts]

4(a) The buoyant acceleration experienced by a parcel of virtual potential temperature θ_v is

$$a = g(\theta_v - \theta_{v,\text{env}})/\theta_{v,\text{env}}$$

where g is the acceleration due to gravity and $\theta_{v,\text{env}}$ is the environmental virtual potential temperature at the same level as the parcel. Assume this to be 300 K in the PBL. The work done against a buoyant force is equal to the force multiplied by the distance moved. Now, consider moving a parcel of air of mass M from just above the inversion atop the PBL down to the earth's surface [this would be like trying to submerge a plastic duck in a bathtub]. Assume that the virtual potential energy of the above-inversion air is 5 K larger than that in the PBL, and that the boundary layer is well-mixed, **calculate the work done in Joules per kilogram of air** assuming the PBL depth is 1500 m.

(b) This process of moving air from above the PBL into the PBL is called *entrainment*, and is an important process which allows boundary layers to deepen by incorporating buoyant free-tropospheric air from above the boundary layer. The rate of deepening of the PBL

through entrainment on a day with reasonably strong surface heating is typically of order 5 cm s^{-1} .

Estimate the rate of mass of air entrained from the free-troposphere per m^2 assuming that the air density is 1 kg m^{-3} . From your answer to (a), what is the rate of working (in W m^{-2}) to bring free-tropospheric air into the PBL?

Unit mass entrained per m^2 column of air = entrainment rate * density. Assume a reasonable value for density (1 kg m^{-3} is fine). Then mass entrained per m^2 is $0.05 \text{ kg s}^{-1} \text{ m}^{-2}$

How does this number compare with the sensible heat flux at the surface from problem 2? What can you conclude about the efficiency of entrainment in general. [3pts]