1. Statistical Quantities in Uncertainty

repeated set of measurements, set of values $\{x_i\}$

↳ distributed about a mean, $\bar{x}$

arithmetic mean (or median, mode, geometric mean)

$$\bar{x} = \frac{1}{N}\sum_{i}^{N} x_i$$

spread about mean (aka distribution) is quantified as average deviation from mean : $d_i = x_i - \bar{x}$

When random fluctuations dominate
average of $d_i \sim 0$

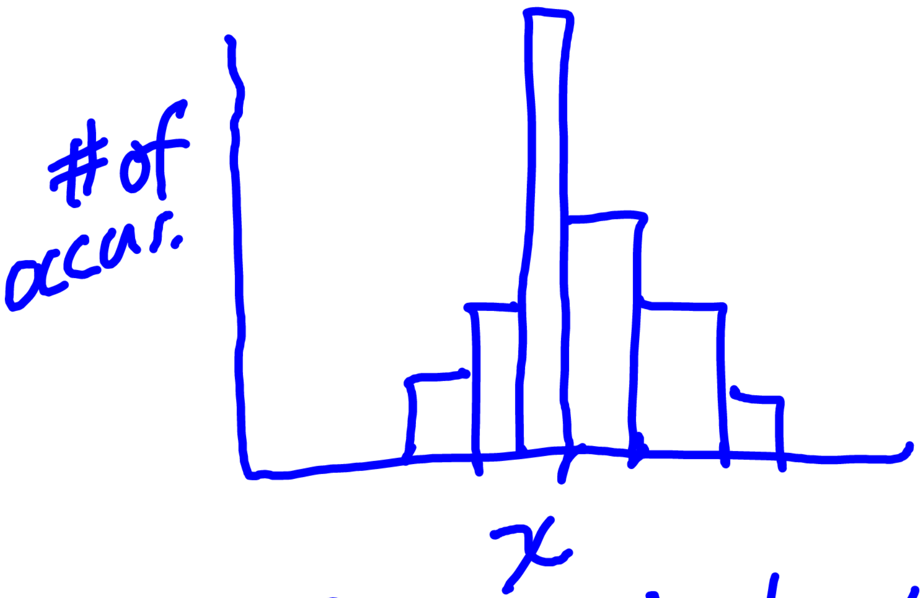
↳ use mean square deviation
$\equiv$ variance $= \sigma^2$

$$\sigma^2 = \frac{1}{N-1}\sum_i^N (x_i - \bar{x})^2$$

↳ from this get root mean square deviation
"standard deviation"
→ same units as the measured quantity

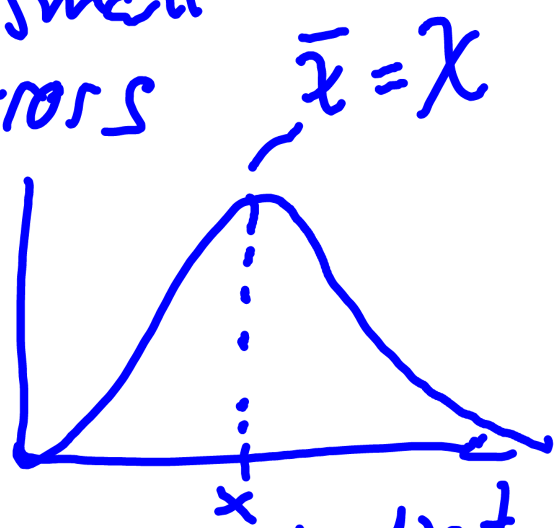
$$\sigma = \sqrt{\frac{1}{N-1}\sum_i^N (x_i - \bar{x})^2}$$

2. LIMITING DISTRIBUTIONS



make many measurements & plagued by small random errors

↳ f



"normal distrib aka Gaussian"

Several distributions exist:

1) normal (Gaussian)
2) binomial
3) Poisson, counting events
4) Lorentz skewed to infrequent but large deviations

Gaussian: $f(x) = e^{-\left(\frac{x^2}{2\sigma^2}\right)}$, symmetric about $x=0$

$$f(x) = e^{-\frac{(x-\bar{X})^2}{2\sigma^2}}$$, symmetric about $x=\bar{X}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{X})^2}{2\sigma^2}}$$, normalized so area $= \underline{1}$

i.e. $\int_{-\infty}^{\infty} f(x)\,dx = \underline{1}$

<u>Two important Points</u>

when $N \to \infty$ & only random errors

1) Mean of distribution is the <u>true value</u>

& thus $\bar{x}$ is a good "best estimate"

$$\bar{x} = \int_{-\infty}^{\infty} x\, f(x)\, dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{(x-\bar{X})^2}{2\sigma^2}} dx = \bar{X}$$

2) The width parameter of the distribution is just variance in the measurements

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x-\bar{x})^2 f(x)\, dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\bar{x})^2 e^{-\frac{(x-\bar{X})^2}{2\sigma^2}} dx$$

$$= \left(\overline{x - \bar{x}^2}\right)$$

$\Rightarrow$ width of normal distribution is quantifying mean square deviation about mean + thus magnitude of <u>random errors</u>.

3. Important TAKE HOME POINTS

1) Above equalities hold only for infinite # of measurements & only "small" random errors

2) Standard deviation of measured values approaches magnitude of all random errors & thus is measure of precision

→ quantify random errors by repeated measurements

3) Standard deviation, σ, represents probable range in which the true value

results

↳ σ represents good measure of uncertainty