Error of MEAN & CONFIDENCE INTERVALS

Example

Town A $\overline{T}_A$ 12.48 °C

Town B → $\overline{T}_B \sim 11.82$ °C

$\overline{T}_B < \overline{T}_A$ by 0.7 °C which is $> \sigma_T$

- Town B is colder (by more than uncertainty)
- NEITHER : uncertainty in difference?

$$\sigma_{\Delta T}^2 = \sigma_T^2 \left(\frac{\partial \Delta T}{\partial T_A}\right)^2 + \sigma_T^2 \left(\frac{\partial \Delta T}{\partial T_B}\right)^2$$

$$\sigma_{\Delta T} = \sqrt{2}\,\sigma_T$$

let x be a measured qnty, make N measurements $\rightarrow \bar{x}$, & σ_x

- $\bar{x}$ is best estimate, intuitively if make more measurements $\rightarrow$ "more confident in $\bar{x}$"
- but more measurements doesn't change σ_x much, implying no more certainty
  - ↳ seems to be a conundrum
- Use E.P.F. to determine uncertainty in mean value
  - N measurements of normally distributed qntity x, $x_1, x_2, \ldots x_N$

$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$; repeat multiple times, $\rightarrow \{\bar{x}_{N,j}\}$ also normally distributed & the width parameter of that distribution, measure of uncertainty in an individual determination of $\bar{x}$

$\hookrightarrow \sigma_{\bar{x}}$

$$\sigma_{\bar{x}}^2 = \sigma_{x_1}^2\left(\frac{\partial \bar{x}}{\partial x_1}\right)^2 + \sigma_{x_2}^2\left(\frac{\partial \bar{x}}{\partial x_2}\right)^2 + \ldots + \sigma_N^2\left(\frac{\partial \bar{x}}{\partial x_N}\right)^2$$

$$\sigma_{x_1}^2 = \sigma_{x_2}^2 = \sigma_{x_N}^2 \equiv \sigma_x^2$$

$$\frac{\partial \bar{x}}{\partial x_i} = \frac{1}{N} \quad \left[\bar{x} = \frac{1}{N}(x_1 + x_2 + x_3 + \dots x_N) \right]$$

$$\Rightarrow \sigma_{\bar{x}}^2 = \sigma_x^2 N \left(\frac{1}{N}\right)^2 = \frac{\sigma_x^2}{N}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$ } standard deviation of the mean or standard error

↳ better usage

- shows benefits of replicate measurements

$\bar{x} \pm \sigma_{\bar{x}}$ means reasonably confident

that the true value lies w/in
$\pm\sigma_{\bar{x}}$ about $\bar{x}$

CONFIDENCE INTERVALS

Area under a normalized Gaussian represents probability that a value will be measured w/in limits used for area

↳ σ_x is natural choice for defining limits of integration

$$P = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\sigma}^{\sigma} e^{\frac{(x-\bar{x})^2}{2\sigma^2}}\,dx = 0.68$$

"$\mathrm{erf}(\sigma)$" error function

$P(2\sigma) = 0.954$