Week 2

Statistical measures of uncertainty

Error Propagation

Temperature Laboratory

Error Propagation (Example)

- You need to know the area of a rectangular plot of land to calculate a moisture flux. You measure the length to be *l* +/- e_l and the width to be *w* +/- e_w.

- What is the best estimate of the ***area, a***, and the associated uncertainty, $\boldsymbol{e_a}$?

Error Propagation Formula (EPF)

For a property *x*, which is a function of measured quantities u, v, and w, i.e., x = f(u,v,w)

$$\sigma_x{}^2 \sim \sigma_u{}^2 \left(\frac{\partial f}{\partial u}\right)^2 + \sigma_v{}^2 \left(\frac{\partial f}{\partial v}\right)^2 + \sigma_w{}^2 \left(\frac{\partial f}{\partial w}\right)^2 + 2\sigma_{uv} \frac{\partial f}{\partial u}\frac{\partial f}{\partial v} + \ \ldots$$

If errors in u, v, and w are *small and uncorrelated*, then

$$\sigma_x{}^2 \sim \sigma_u{}^2 \left(\frac{\partial f}{\partial u}\right)^2 + \sigma_v{}^2 \left(\frac{\partial f}{\partial v}\right)^2 + \sigma_w{}^2 \left(\frac{\partial f}{\partial w}\right)^2$$

EPF Application Examples: Find σ_x^2

- $x = u + a$; a is a constant
- $x = au + bv$; a and b are constants
- $x = auv$; a is a constant
- $x = uv/(wz)$

Announcements

- Quiz on Friday 1/22 in class
- Weather station data discussion
- FULL draft report on T laboratory DUE Thursday 1/28

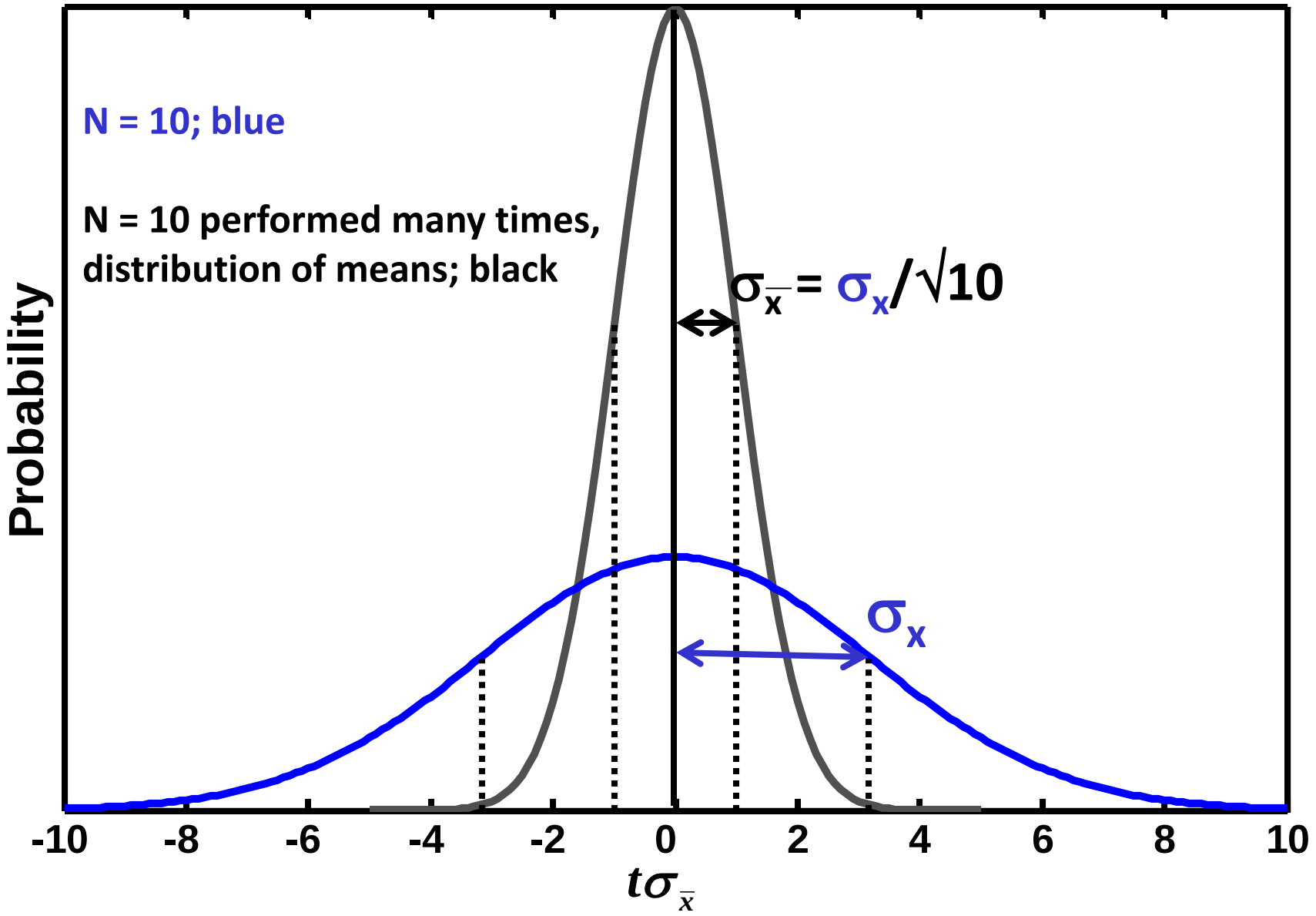
Question

Two *weathernuts* living in adjacent towns (town **A** and **B**) measure the air temperature in their respective town during a brief period. Both want to claim their town was colder than the other during this time. Does either one have a valid claim?

Town A *T* Measurements: 10.2, 11.5, 13.4, 15.1, 12.2 ^{o}C
Town B *T* Measurements: 9.8, 10.2, 12.8, 14.6, 11.7 ^{o}C

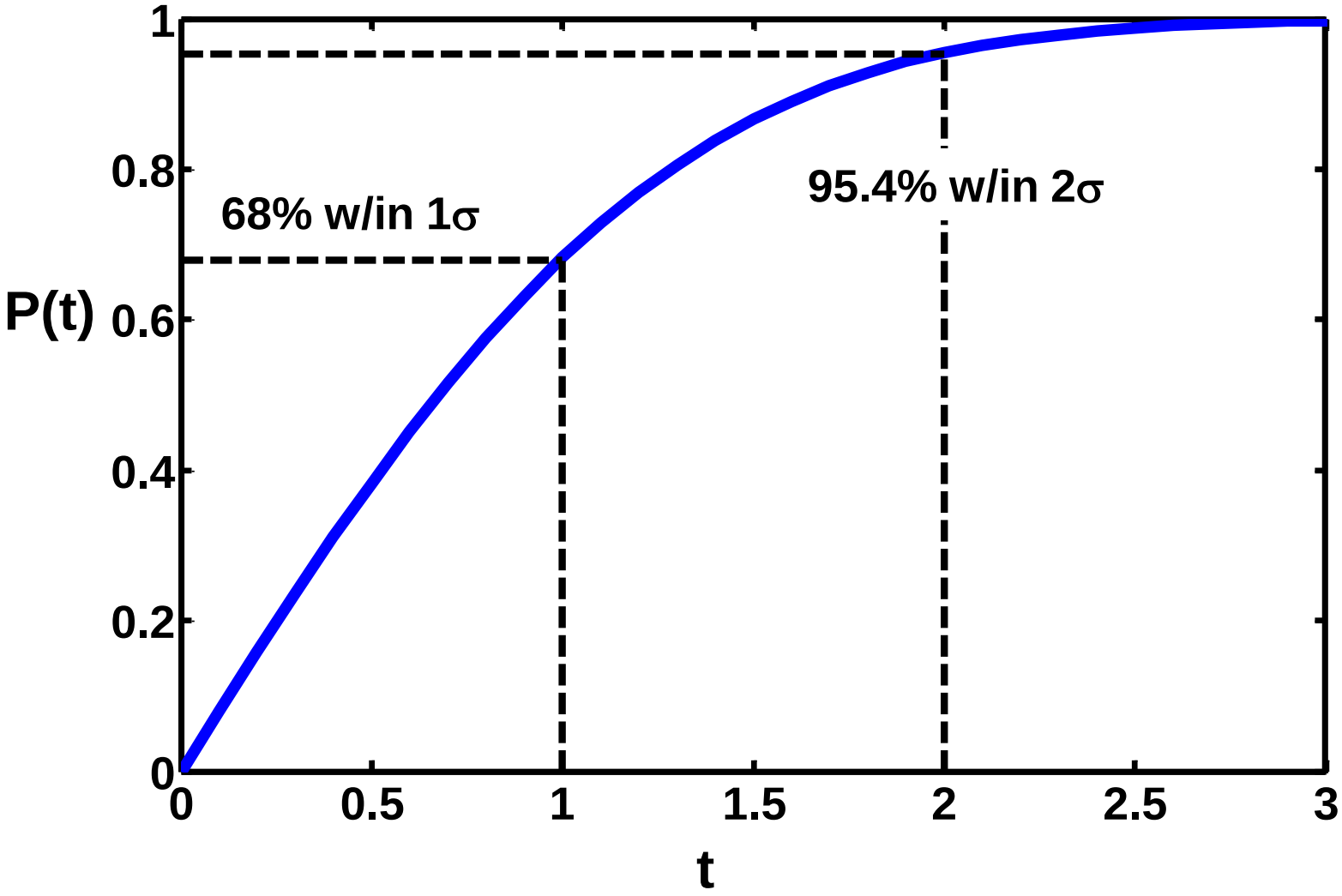
The uncertainty in any *one* of weathernut A's or weathernut B's individual measurements = 0.5 ^{o}C.

Distribution of *N* Measurements and of Means

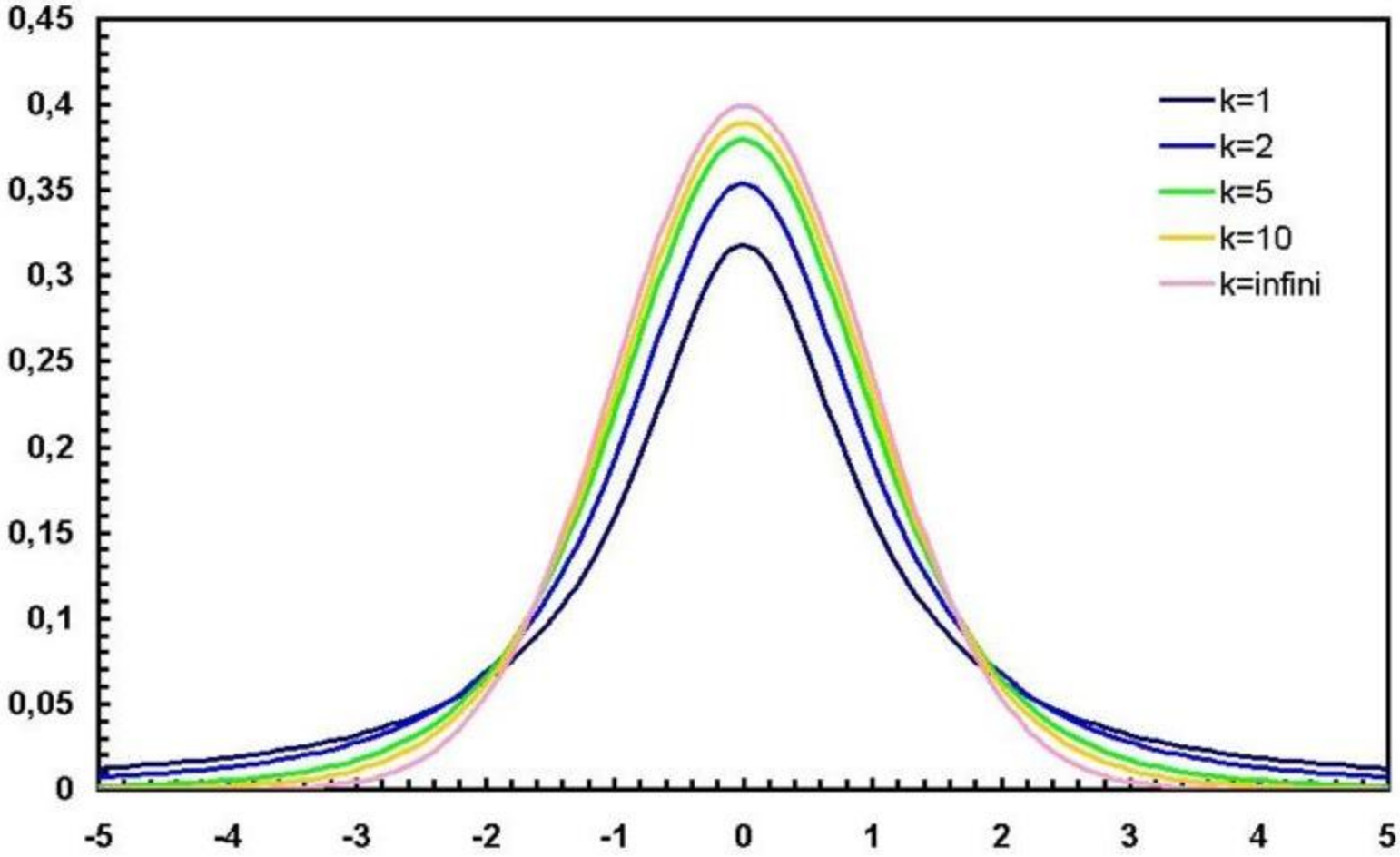


Normal Error Integral

$$P(t) = \frac{1}{\sqrt{2\pi}} \int_{-t\sigma}^{t\sigma} e^{\frac{-z^2}{2}} dz$$



Normal vs Student's t-distribution



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