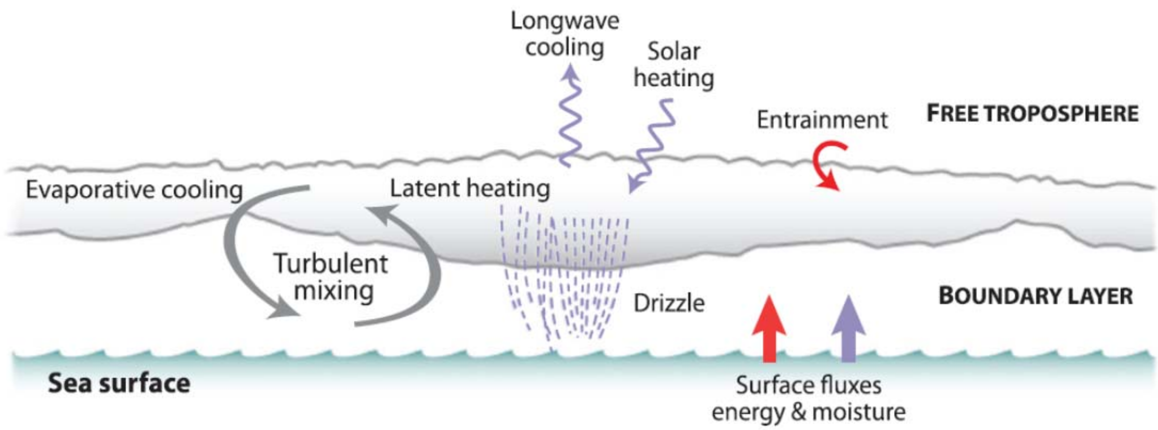
Cloud-topped mixed layer

- Stratocumulus-topped boundary layers [STBL] dominant cloud type over oceans [24% coverage over ocean; 12% over land].
- Stratocumulus often exists within a mixed or quasi-mixed layer in which conserved variables are constant with height, capped by strong inversion
- Several key cloud properties can be predicted using mixed layer theory

Key processes



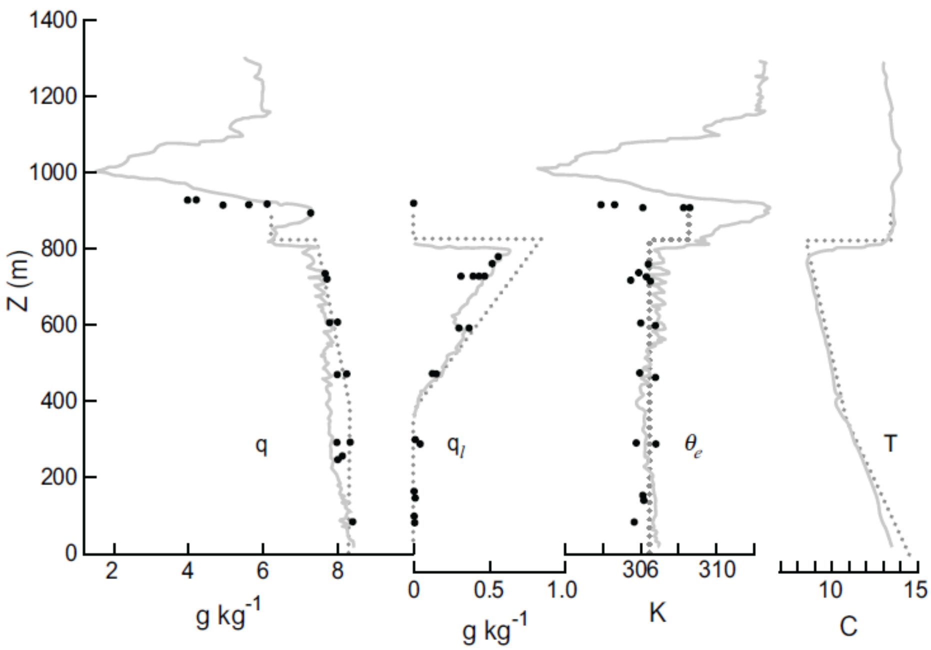
Schematic showing the key processes occurring in the stratocumulus-topped boundary layer

- **Longwave cooling** – key driver of buoyant instability in the STBL by cooling upper 10s of meters of cloud. The free troposphere (FT) is often very dry and transmissive in IR allowing ~100 W m^{-2} of net LW flux at top of cloud layer. During daytime, IR cooling is offset by SW warming, suppressing turbulence generation
- **The surface flux of sensible heat** is often quite small, especially over oceans
- **Latent heat flux** from surface is important in maintaining moisture transport into cloud layer and for creating latent heat release to drive buoyant thermals and turbulence kinetic energy (TKE) generation
- **Cloud top entrainment** is driven by turbulent impingement and erosion of stable inversion. Mixes warm, dry air from the FT into the STBL. Is a primary sink of TKE in the STBL. Is assisted by **evaporative cooling** of mixed cloudy-clear parcels.
- **Drizzle production** can be important in some (not all) STBLs. Most prevalent over the ocean and in thick stratocumulus (>300 m) where droplet concentrations are low and cloud drops reach sufficient size to favor significant collision-coalescence. Drizzle is another important sink of TKE for the STBL as a whole (warming cloud layer; cooling subcloud layer). However, evaporation of drizzle in the subcloud layer can be a significant source of TKE generation there, forming cold pools that have a profound impact on the mesoscale organization.

Mixed layer theory

Variables: What variables are conserved under moist adiabatic processes?

- Equivalent potential temperature θ_e – or, alternatively, moist static energy ($S = c_p T + gz + Lq_v = c_p\theta + Lq_v$)
- Total water (vapor+condensate) mixing ratio q_T.

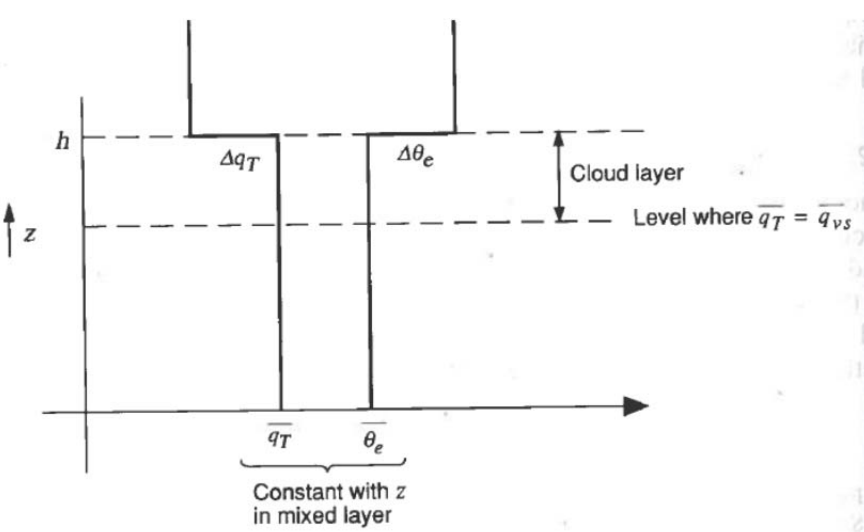


Stratocumulus-topped boundary layer profiles. From Nicholls (Q. J. Roy. Meteorol. Soc., 1984)

Cloud layer: In the mixed layer, the part of the layer where $q_T > q_s$ is saturated and is therefore the cloud layer. **Cloud thickness** is determined by the distance between the mixed layer top, height h, and the lifting condensation level (LCL) where $q_T = q_s[T(z), p(z)]$.

Figure: Assumed variation of q_T and θ_e with height in the mixed layer. Δx is the jump at the top of the mixed layer in variable x

From Houze: Cloud Dynamics (1993)



Mixed layer budget equations:

The budget equation for the layer-mean value $\overline{q_T}$ of the total water mixing ratio is:

$$\frac{d\overline{q_T}}{dt} = -\frac{\partial}{\partial z}\left(\overline{w'q_T'} - P\right) \quad [1]$$

where $\overline{w'q_T'}$ is the turbulent flux of total water mixing ratio, and P is the precipitation rate (defined positive for downward flux). Similarly, for moist static energy:

$$\frac{d\overline{\theta_e}}{dt} = -\frac{\partial}{\partial z}\left(\overline{w'\theta_e'} + R\right) \quad [2]$$

where $\overline{w'\theta_e'}$ is the turbulent flux of equivalent potential energy, and R is the net radiative flux.

Finally, the mass budget of the PBL can be expressed as:

$$\frac{dh}{dt} = w_e - w_s(h) \quad [3]$$

where h is the layer depth, w_e is the cloud top entrainment rate, and $w_s(h)$ is the large scale subsidence rate.

Boundaries: The surface and the interface with the FT are sources and sinks for q_T and θ_e.

Surface: $(\overline{w'q_T'})_0$ is the surface flux of total water = E/ρ where E is the evaporation rate [kg m^{-2}], and ρ is the air density. It is thus directly related to the latent heat flux. The surface flux of θ_e is related to the sum of the latent and sensible heat fluxes, expressed in the units of K m s^{-1} using the conversion factors L and c_p respectively, using $c_p\theta_e' \approx c_pT' + Lq_v'$

Cloud top: Turbulent fluxes associated with entrainment at the top of the mixed layer can be written as

$$\left(\overline{w'q_T'}\right)_h = -w_e\Delta q_T \quad [4]$$

$$\left(\overline{w'\theta_e'}\right)_h = -w_e\Delta\theta_e \quad [5]$$

where w_e is the entrainment rate (rate in m s^{-1}) at which FT air is brought into the PBL), and Δx represents the jump at the top of the mixed layer in a variable x. If the contrast between the mixed layer and the FT is larger, then, for a given value of w_e the flux will be larger.

Mixed layer constraint:

In order for the PBL to remain well mixed, dq_T/dt must be independent of height over the depth of the mixed layer (otherwise one level would increase at a different rate from another level, therefore breaking the constraint of there being a well-mixed layer). Therefore, Eqns. [1] and [2] can be written:

$$\frac{d\overline{q_T}}{dt} = -\frac{\partial}{\partial z}\left(\overline{w'q_T'} - P\right) = \text{constant} \ \ (0 < z < h) \qquad [6]$$

where $\overline{w'q_T'}$ is the turbulent flux of total water mixing ratio, and P is the precipitation rate (defined positive for downward flux). Similarly, for moist static energy:

$$\frac{d\overline{\theta_e}}{dt} = -\frac{\partial}{\partial z}(\overline{w'\theta_e'} + R) = \text{constant} \ \ (0 < z < h) \qquad [7]$$

Therefore, the terms inside the parentheses on the RHS of [6] and [7] are linear with height, and thus simply given by the difference between the fluxes at the top and the bottom of the layer divided by the layer thickness. We assume zero precipitation at the layer top and a non-zero value P_0 at the surface:

$$\frac{d\overline{q_T}}{dt} = \left[\frac{w_e\Delta q_T + (\overline{w'q_T'})_0 - P_0}{h}\right] \qquad [8]$$

$$\frac{d\overline{\theta_e}}{dt} = \left[\frac{w_e\Delta\theta_e + (\overline{w'\theta_e'})_0 - R_h + R_0}{h}\right] \qquad [9]$$

Entrainment closure

Now, given known values of the radiative fluxes and the precipitation, the three equations [3], [8] and [9] representing the mass, q_T and θ_e budgets have four unknowns[1] h, $\overline{q_T}$, $\overline{\theta_e}$, and w_e. There is, therefore, a need to provide an additional constraint (called a "closure") on one of these variables. Typically, this is carried out by representing the cloud top entrainment rate w_e in terms of the other variables. A complete treatment of how this is done is beyond this class, but a conceptual sense can be provided by thinking of entrainment as doing work against the buoyancy gradient between the mixed layer and the FT (FT air being lighter). One set of argues that some specified fraction of the TKE generation goes into entrainment work. The more TKE production there is, the more available TKE there is to produce entrainment.

Fortunately, the mixed layer equations actually provide the means to estimate buoyant TKE production. This was the genius of the seminal paper on the mixed layer by Lilly (*Quart. J. Roy. Meteorol. Soc.*, 1968). The equations [6] and [7] provide profiles of the turbulent fluxes of q_T and θ_e (i.e. $\overline{w'q_T'}$ and $\overline{w'\theta_e'}$ respectively) required to keep the PBL mixed. A linear combination of these fluxes (see e.g. Bretherton and Wyant., *J. Atmos. Sci.*, 1997) is sufficient to provide an estimate of the turbulent flux of virtual temperature – this is the buoyancy flux which is the rate of buoyant TKE production. This is complicated somewhat by the fact that in general, the mixed

[1] Bear in mind that the surface fluxes in q_T and θ_e can be represented using bulk flux formulae (see e.g. Hartmann notes) in terms of the mean values of $\overline{q_T}$ and $\overline{\theta_e}$ and the surface temperature and wind speed.

layer contains a portion that saturated such that liquid loading also contributes to the buoyancy flux and latent heat release contributes to the virtual temperature.

Role of radiation

One can see that the net radiation term $-R_h + R_0$ in [9] represents the net radiative flux divergence over the mixed layer. Typically, for the STBL, R_0 is quite small and R_h is large because a considerable LW flux leaves the top of the mixed layer (emission by cloud droplets) while a much smaller downwelling LW flux enters. Even without clouds, there is a significant net flux divergence from the PBL due to emission from water vapor in the PBL. Thus the net radiation effectively drives the PBL towards saturation and helps maintain it in this state once the layer has become saturated. Radiation is also critical for cooling parcels so that they sink and generate TKE (see Schubert circuit diagram figure below). Daytime offsetting of the LW cooling by SW absorption will reduce the overall cooling experienced by a parcel near cloud top and reduce the overall TKE generation.

Precipitation

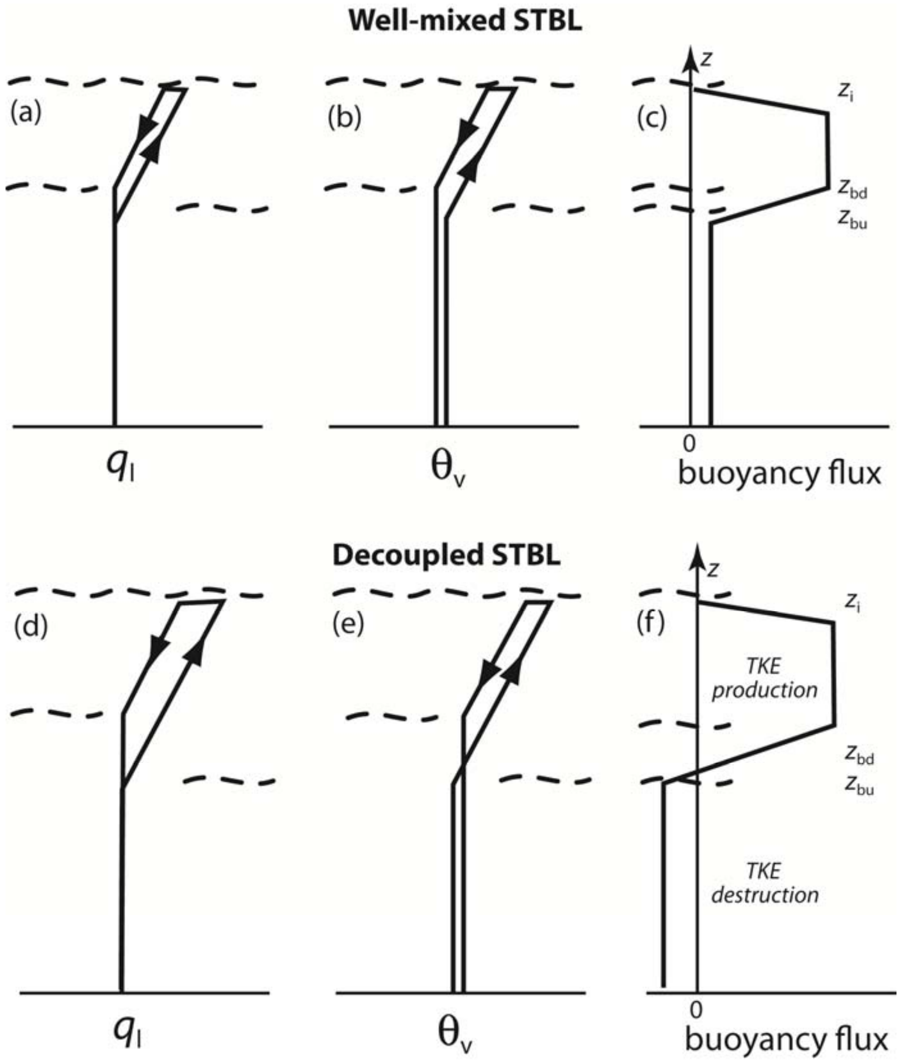
Precipitation has two important effects: (1) reduction of total water mixing ratio in the mixed layer, i.e. Equation [8], which *ceteris paribus* (all else being equal) will lift the LCL and thin the cloud; (2) effect on buoyancy. Removal of liquid water from the parcel as it ascends will raise the LCL for the descending branch of the circuit (see Schubert diagram) and tend to lead to a situation where there is negative buoyancy in the descending parcel around cloud base, a loss of buoyant TKE below cloud base and a tendency for the PBL to decouple.

Effect of free-tropospheric conditions

A very dry FT will tend to drive strong evaporative cooling at the mixed layer top, and will also promote stronger LW cooling. Both favor greater TKE generation by increasing cooling experienced by parcels near cloud top, but the effect of the former is complex because it also removes condensate from the downdraft (think about the Schubert diagram) which can promote decoupled. Stronger evaporation can also drive more efficient entrainment which may offset much of the evaporative cooling.

A warm FT favors a lower entrainment rate because for a given buoyancy flux at cloud top there will be less entrainment rate if the stratification is strong. This tends to favor a shallower mixed layer (Eqn. [3]), which may remain well-mixed instead of decoupling.

Schubert parcel circuit diagrams illustrating the path of an idealized parcel undergoing circulations in the STBL.

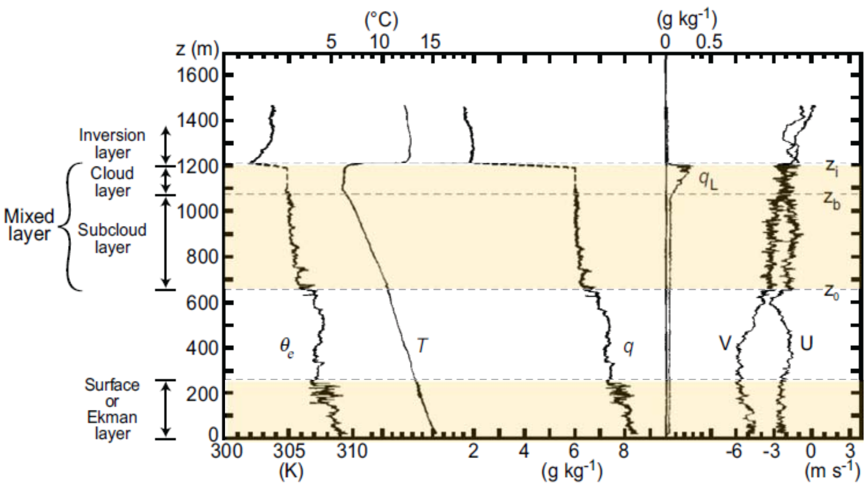


Top: Well-mixed STBL. Consider a parcel below cloud moving upward. Its virtual potential temperature will remain constant until cloud base where latent heat will warm the parcel at the moist adiabatic lapse rate (center). Liquid water mixing ratio q_l (left) for the ascending parcel increases with height. Entrainment of dry air into parcel at the STBL top evaporates some liquid so that the descending parcel has lower q_l at the same level. Although entrainment warms the parcel, radiative and evaporative cooling take place and cool the parcel until it begins to sink. It reaches its LCL and then, provided it is negatively buoyant at that level, will continue to sink. At all levels, the downward moving parcel is virtually cooler than the upward moving parcel, so there is buoyant TKE generation at all levels. The PBL can remain well mixed.

Bottom: Decoupled STBL. Entrainment has removed sufficient liquid water and/or there was insufficient LW or evaporative cooling, such that the downward moving parcel at cloud base is warmer than the upward moving parcel and there is TKE destruction throughout the subcloud layer.

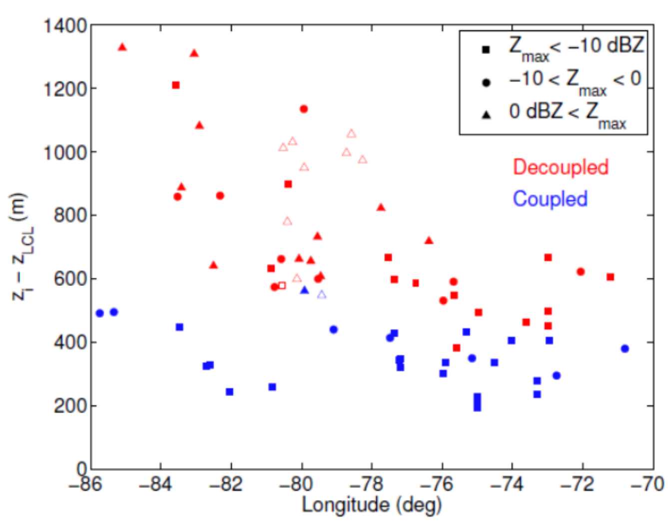
Evolution and decoupling of the STBL

Observations demonstrate many examples of well-mixed STBLs all over the oceans (e.g. Figure from Nicholls) and even over land areas. One important condition that seems to be common to well-mixed STBLs is that they tend to be shallow. Most are shallower than 1 km, although exceptions are found with well-mixed layers up to 1.5 km deep. The figure below shows a typical set of profiles for an STBL that is not well mixed. There is clearly identifiable stratification in the conserved variables q_T and θ_e. Whereas the cloud layer is existing within a layer that is well-mixed, the mixed layer is elevated and disconnected (decoupled) from the surface. It turns out that the term *decoupled* is not entirely accurate. It is more appropriate to say that the STBL is intermittently-coupled, or *cumulus-coupled* (because patchy cumulus clouds extending up from the top of the surface layer serve as the primary communicators of information, energy, moisture, etc. between the surface and the stratocumulus-containing mixed layer aloft.



Example profile from a decoupled STBL. The upper yellow box indicates the extent of the stratocumulus-containing mixed layer and the lower box is the surface mixed layer

From Nicholls and Leighton Quart. J. Roy. Meteorol. Soc. 1986)



Decoupled vs coupled STBLs vs longitude over the southeastern Pacific Ocean. Here, coupled indicates well-mixed and decoupled indicates less well mixed. From Jones et al. (2012, Atmos. Chem. Phys.)

Why is the PBL height an important factor controlling decoupling?

For a layer to remain well-mixed sufficient TKE must be generated to sustain the losses of TKE by dissipation. Mostly TKE in the STBL is produced by buoyant production, i.e. updrafts that are positively buoyant with respect to downdrafts.

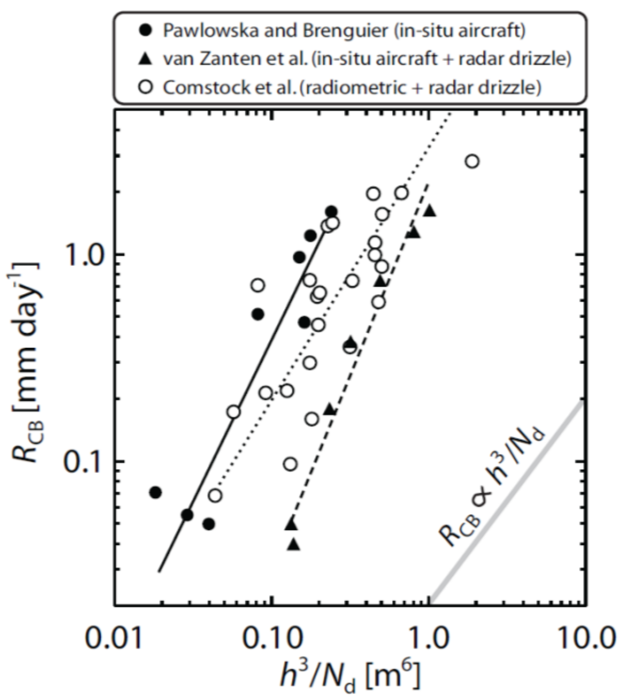
As the STBL mixed layer deepens, one or both of the following will occur: the cloud will thicken; the surface moisture flux will increase (because a higher LCL necessarily means lower surface RH and greater evaporation for a given wind speed). Both will promote greater TKE generation by increasing the overall latent heat release.

However, both drive a greater entrainment rate, which will cause greater evaporation of liquid at the cloud top and a greater difference in the height of the LCL for upward and downward moving parcels (see the lower row of the Schubert circuit diagram above). This tends to produce downdrafts that have a greater tendency to be positively buoyant compared with the neighboring updrafts, and so will consume TKE and, eventually, not reach the surface.

Factors contributing to deepening STBLs: increased SST (cold subtropics to warm tropics), reduced subsidence, reduced stability [by increasing entrainment].

Warm rain in STBLs

A key factor controlling precipitation is the availability of liquid water for accretion onto embryonic raindrops. Thus, it is not surprising that the primary factor determining whether stratocumulus precipitates is the cloud thickness (since this, for a mixed layer, determines the condensate profile). However, as we have seen, the droplet size is also important because the collection efficiency drops sharply for droplets with radii smaller than ~10 μm. A dependence of precipitation rate on cloud *microphysical* properties (in addition to a strong dependence upon cloud thickness) can be seen in observations (see figure).



Precipitation rate at cloud base from different studies of stratocumulus clouds, plotted against the ratio of the third power of the cloud thickness to the cloud droplet concentration. From Wood (2012, *Mon. Wea. Rev.*)

Factors controlling cloud droplet concentration

As we have seen, quantifying the number concentration N_d of cloud droplets formed upon the ascent of a parcel requires a prognostic treatment of the supersaturation equation (see figure below). In general, the majority of cloud droplets are formed on accumulation mode aerosol (sizes from 50-500 nm), and so N_d tends to increase with the accumulation mode concentration N_a. However, in generating more droplets, increased N_a leads to a greater sink for supersaturation and a reduction in the maximum supersaturation experienced by the ascending parcel *ceteris paribus*. This means that the minimum size above which aerosols will activate will increase and that the gain from increasing N_a will be somewhat muted. Thus, typically, N_d is a somewhat concave increasing function of N_a (see figure below)

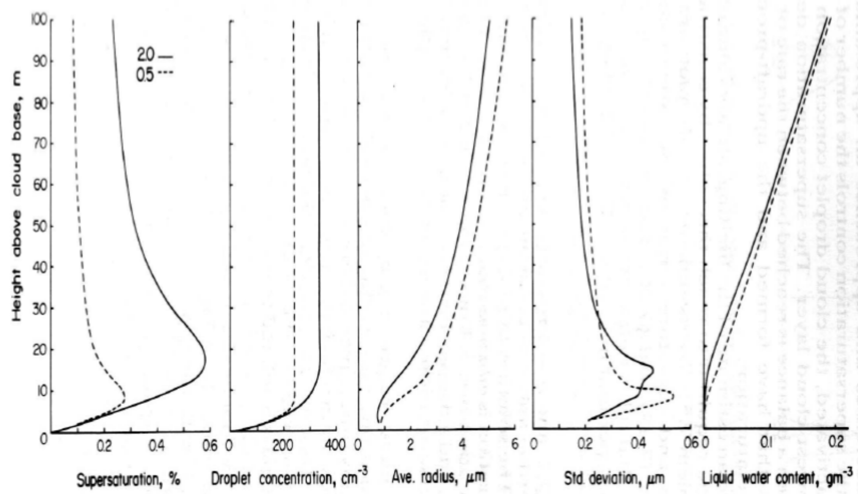


FIG. 7.4. Early development of cloud properties in air ascending at constant velocity of 0.5 m/s or 2 m/s.

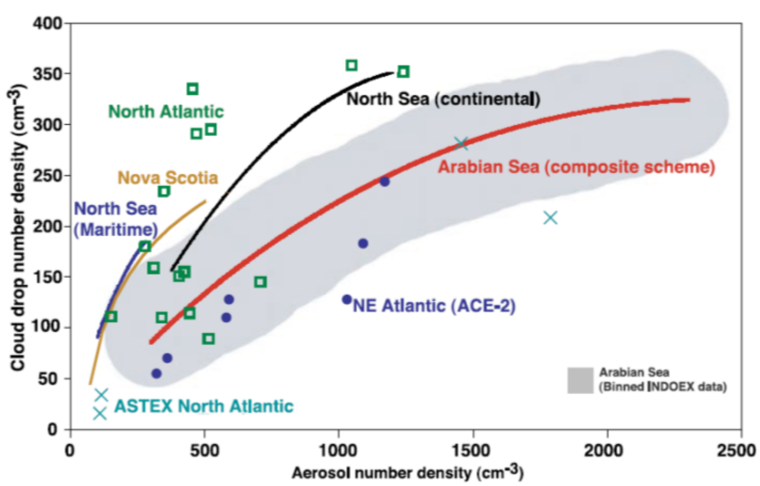
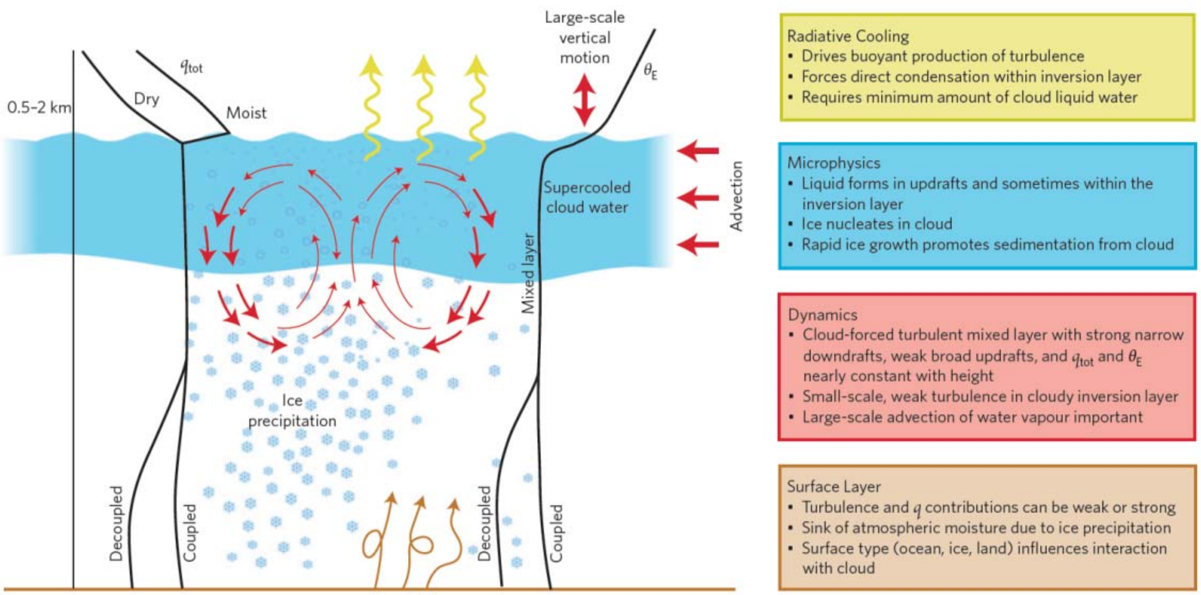


Fig. 5. Aircraft data illustrating the increase in cloud drops with aerosol number concentration. References for the data are as follows: North Sea (28), Nova Scotia and North Atlantic (29), ACE-2 (30), Astex (31), the thick red line is obtained from a composite theoretical parameterization that fits the INDOEX aircraft data for the Arabian Sea (23). The gray-shaded region is the INDOEX aircraft data for the Arabian Sea (32).

Left, from Ramanathan et al., (*Science*, 2001)

Cold-topped STBLs

In many ways, cold-topped STBLs behave very similarly to warm-topped ones. One key difference is the fact that ice may be present if the cloud top temperature is sufficiently cold. For STBLs over sea-ice, the surface can be much colder than the atmosphere and so the STBL may be frequently decoupled by a strong near-surface inversion from the ice below. In these cases, there is no surface moisture source to offset losses due to precipitation. However, observations show that the FT can be moister than STBL in many cases, so another key difference between Arctic and subtropical STBLs is that in the former, entrainment may actually provide a source rather than a sink of moisture.



- Altocumulus behavior is very similar to that of stratocumulus, and may exist within a mixed layer driven by LW cooling. Over time, the layer may glaciate and the layer can be dissipated in this way.