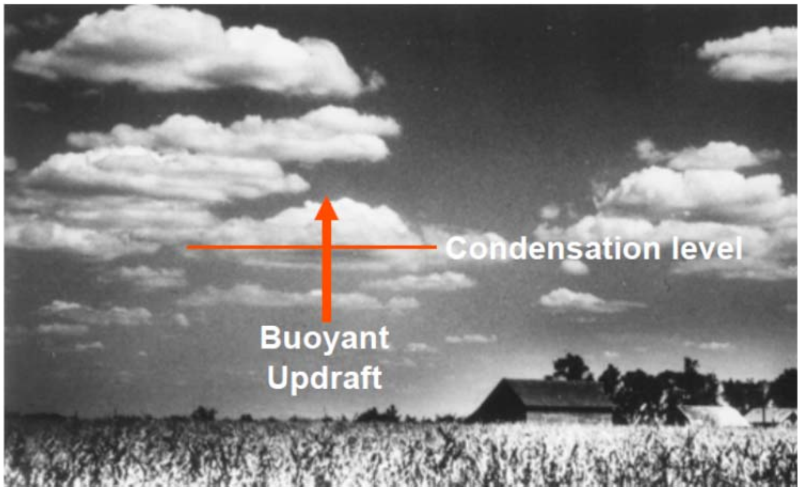
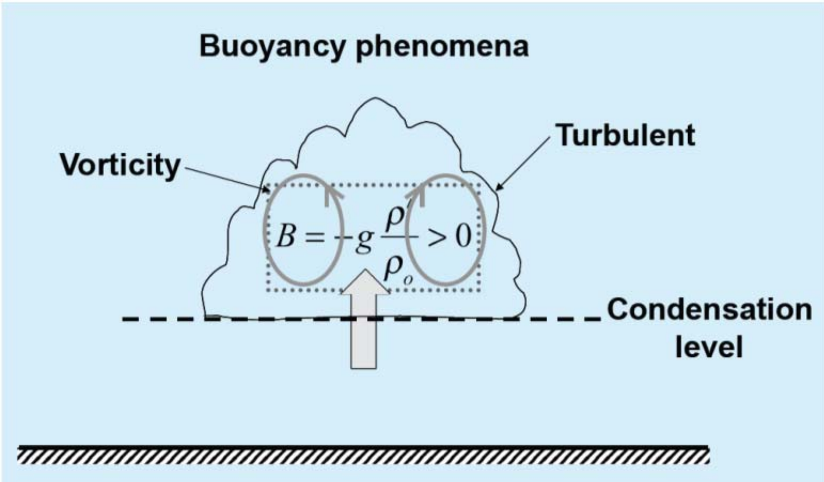
Cumulus clouds

- Form under conditions of buoyant instability, usually surface driven
- Different forms: fair weather, trade cumulus, congestus, cumulonimbus
- Trade Cu, congestus and cumulonimbus produce precipitation in increasing amounts. Severe cumulonimbus over land can develop hail and tornados
- Some Cu develop anvils - detrained hydrometeors that assume a stratiform nature
- Can aggregate and develop into mesoscale convective systems, open cells, etc. Diabatic effects (e.g. cold pools, radiation) are important for such development

Buoyant updrafts raise parcels above the LCL. Condensation of vapor under conditions of conditional instability leads to upward acceleration of parcels





Positive buoyancy within a cloud system means that the density within the rising parcel is less than that in the environment outside. This picture is appropriate for all cumulus clouds.

In the thermodynamic section, we examined the equation for vertical acceleration:

$$\frac{dw}{dt} = -\frac{1}{\rho_e}\frac{\partial p'}{\partial z} + B = -\frac{1}{\rho_e}\frac{\partial p'}{\partial z} + g\left[\frac{T'}{T_e} + 0.61q_v' - \frac{p'}{p_e} - q_c\right] \quad [1]$$

Where w is the vertical wind speed in the parcel, T_e and p_e are the environmental temperature and pressure at the level of the parcel, T' and p' and q_v' are the perturbations in the plume temperature, pressure, and water vapor mixing ratio compared with the environment, and q_c is the condensate mass.

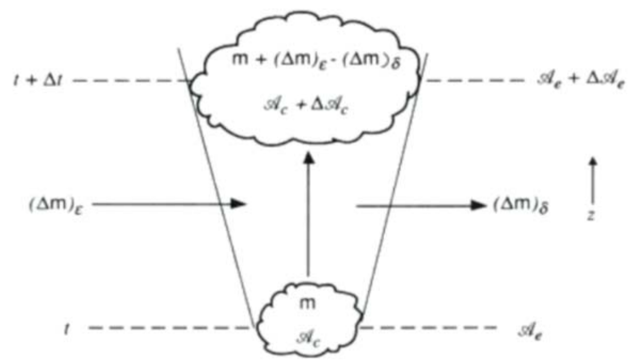
The equation for mass continuity is

$$\nabla \cdot (\rho_e \underline{u}) = 0 \qquad [2]$$

One dimensional Lagrangian parcel model

Ascending parcel subject to vertical equation of motion and continuous, lateral (from the sides) and homogeneous (evenly distributed throughout the parcel) entrainment of environmental air.

Parcel ascending from one level (time) to another at a time Δt later. The parcel mass changes due to the incorporation of environmental air entrained into the parcel, and due to the detrainment of parcel material into the environment. This results in a change $\Delta\mathcal{A}_c$ in any given parcel property $\mathcal{A}_c$ (having units of energy, mass or momentum per unit mass of air)



$$\frac{d\mathcal{A}_c}{dt} = \left(\frac{\partial \mathcal{A}_c}{\partial t}\right)_{noent} + \frac{1}{m}\left(\frac{dm}{dt}\right)_{ent}(\mathcal{A}_e - \mathcal{A}_c) \qquad [3]$$

$(d\mathcal{A}_c/dt)_{noent}$ is the rate of change of $\mathcal{A}_c$ that would occur without the exchange with the surroundings (e.g. that due to latent heat release or other processes).

Moist static energy $h = c_p T + gz + Lq_v$ is conserved for moist adiabatic processes. In the absence of other diabatic processes (e.g. radiation), h is conserved in a parcel, so substituting $\mathcal{A}_c = h_c$ into [3], and noting that $(dh/dt)_{noent}$=0, we have

$$\frac{dh_c}{dt} = \frac{1}{m}\left(\frac{dm}{dt}\right)_{ent}(h_e - h_c) \qquad [4]$$

From the definition of moist static energy, we can obtain an expression for the rate of change of temperature in the parcel:

$$\frac{dT_c}{dt} = -\frac{g}{c_p}w_c - \frac{L}{c_p}\frac{dq_v}{dt} + \frac{1}{m}\left(\frac{dm}{dt}\right)_{ent}\left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right] \quad [5]$$

The terms on the RHS of [5] represent dry adiabatic cooling, latent heating, and the effects of entrainment respectively. Equation [1] can be modified following [4] to include entrainment in the vertical acceleration of the parcel (assuming the environment is stationary):

$$\frac{dw_c}{dt} = -\frac{1}{\rho_e}\frac{\partial p'}{\partial z} + B - \frac{1}{m}\left(\frac{dm}{dt}\right)_{ent} w_c \quad [6]$$

For water mixing ratio, the equation becomes

$$\frac{dq_{vc}}{dt} = -C + \frac{1}{m}\left(\frac{dm}{dt}\right)_{ent}(q_{ve} - q_{vc}) \quad [7]$$

where C is the condensation rate in the parcel.

Typically, instead of couching the derivatives in the time coordinate, we transform to the spatial dimension, as $dz/dt = w_c$. With this transformation, Eqns. 5-7 become

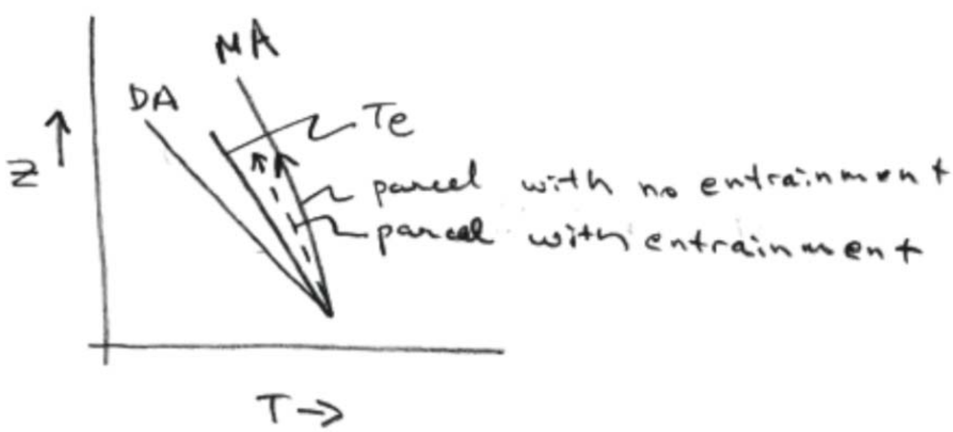
$$\frac{dT_c}{dz} = -\frac{g}{c_p} - \frac{L}{c_p}\frac{dq_v}{dz} + \Lambda\left[(T_e - T_c) + \frac{L}{c_p}(q_{ve} - q_{vc})\right] \quad [8]$$

$$\frac{d\left(\frac{1}{2}w_c^2\right)}{dz} = -\frac{1}{\rho_e}\frac{\partial p'}{\partial z} + B - \Lambda w_c^2 \quad [9]$$

$$\frac{dq_{vc}}{dt} = -\frac{C}{w_c} + \Lambda(q_{ve} - q_{vc}) \quad [10]$$

where $\Lambda = \frac{1}{m}\left(\frac{dm}{dz}\right)_{ent}$ is a measure of the entrainment rate (fractional change in parcel mass per height interval). Equations 8-10 can be solved for the ascending parcel, but this requires the provision of a value for Λ and for the pressure perturbation term in [9].

Temperature change for ascending parcel with and without entrainment



Microphysical processes

Assuming that we have two liquid species (cloud water q_{lc} and rain water q_{rc}), and that condensation of vapor only affects the former, we can apply conservation of water, assuming no condensate outside of the cloud:

$$\frac{dq_{lc}}{dz} = \frac{C}{w_c} - \Lambda q_{lc} - \frac{A_c + K_c}{w_c} \quad [11]$$

$$\frac{dq_{rc}}{dz} = -\Lambda q_{rc} + \frac{A_c - K_c}{w_c} - F_r \quad [12]$$

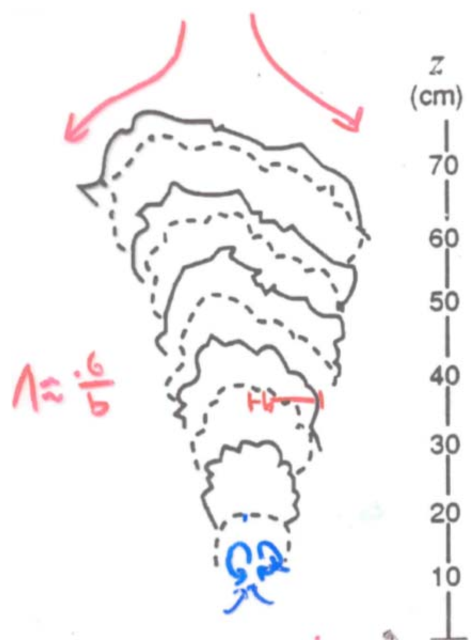
Here, A_c is the autoconversion rate (conversion of cloud drops into raindrops by coalescence) and K_c is the accretion rate (collection of cloud drops by raindrops). F_r is the loss of precipitation by gravitational settling from the parcel. Refer back to the coalescence notes. The bulk cloud model homework explores this in more detail.

Lateral entrainment

The entrainment rate Λ is obtained empirically. Turbulent jets in the laboratory can be used as analogues. In these, it is found that Λ is inversely proportional to the half-width d of the jet, because entrainment scales with the circumference of the jet ($\propto d$) whereas the mass scales with the area ($\propto d^2$). The lateral entrainment closure problem is similar to that involved in the cloud-topped mixed layer problem, and there is ongoing research and debate regarding the complete set of physical processes responsible.

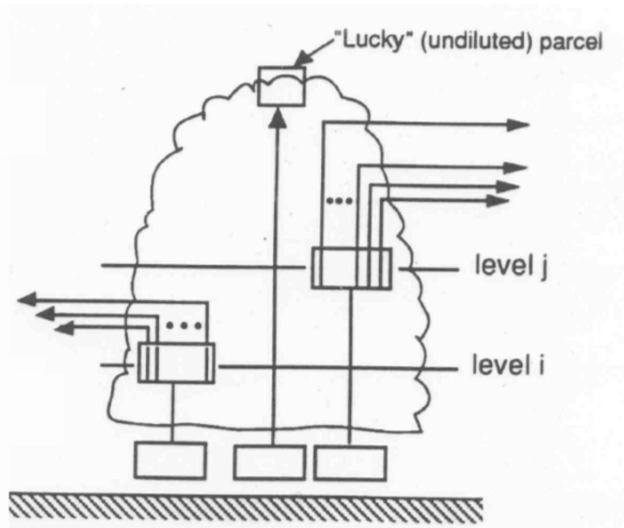
$$\Lambda = \frac{\alpha}{d} \quad [13]$$

where α is an empirical constant (~0.2 in laboratory jets). In real cumulus clouds, the fluid is compressible, and the cloud consists of a series of bubbles (the "bubble model"), evaporative cooling at the cloud edges can assist entrainment, and the bubbles are like small spheres so entrain over their entire surface. These structures are referred to as thermals instead of jets. Laboratory analogues of thermals indicate stronger entrainment typically, such that $\Lambda \sim 0.6/d$.



Outlines of laboratory thermals from photographs.

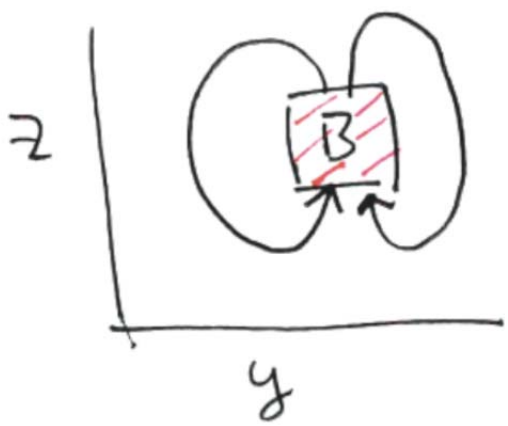
Observations indicate that entrainment does not immediately and continuously mix the environmental air throughout the cloud homogeneously. An alternative model of entrainment is one of intermittent entrainment events that mix only a portion of the cloud. This is a level of added complexity that can address some of the observed microphysical structural features of cumulus clouds, including intermittent regions of largely undiluted condensate punctuated by strongly diluted patches. It is difficult to represent such entrainment within the framework discussed above.

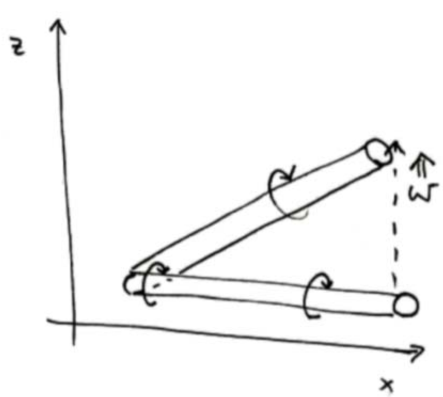


Raymond and Blyth's model of discontinuous, inhomogeneous entrainment

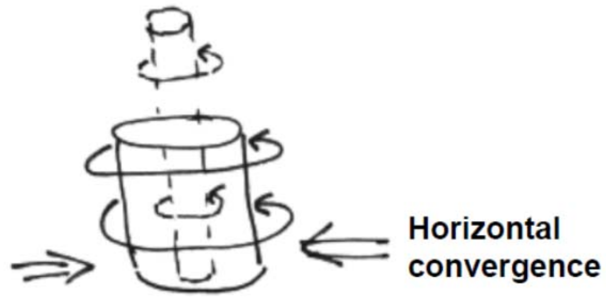
Vorticity

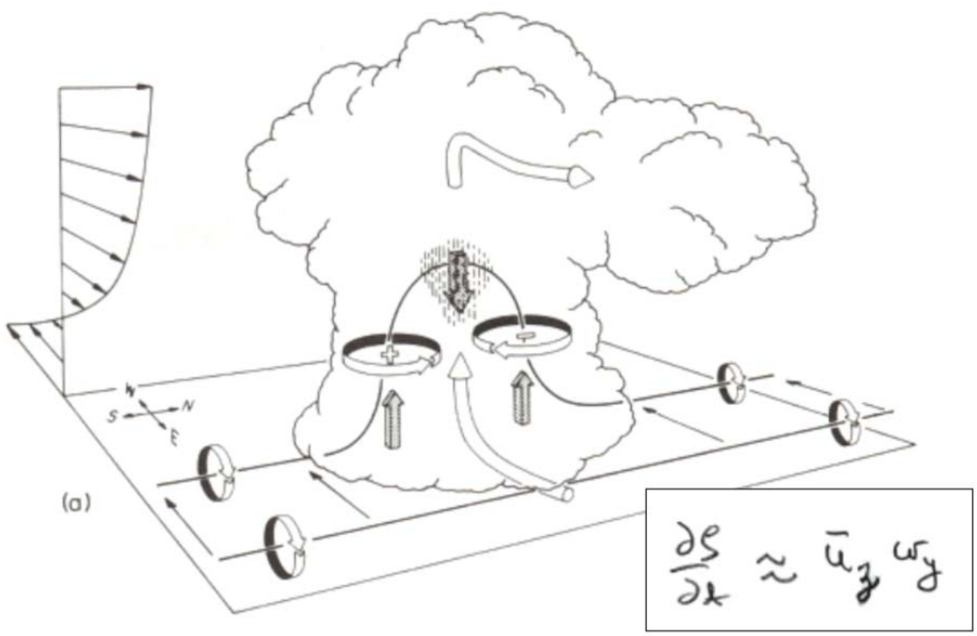
- Convective clouds develop vertical motions
- Simplest example is thermal rising like a smoke ring (right)
- Downwardly accelerating negatively buoyant parcels in downdrafts also generate vorticity ("microbursts")
- When horizontal vortex tubes associated with convective clouds become tilted, some of the rotation becomes about a vertical axis – increased vorticity (the "tilting term", see right)





- Horizontal convergence increases vorticity (the “stretching term”, see right)
- Development of a vorticity couplet (see below) from tilting term in a cloud developing in a sheared environment
- Mesocyclones can create funnel clouds and ultimately tornados via stretching and tilting





Pressure perturbation

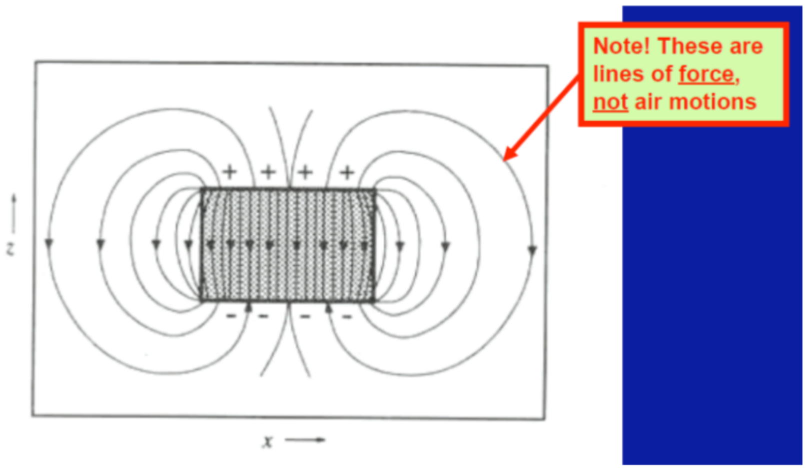
Buoyancy cannot exist without a simultaneous disruption of the pressure field. If a less dense parcel than the surrounding air, then the pressure at the base of the parcel will be less than the environment. The horizontal pressure gradient accelerates environmental air toward the base of the buoyant parcel. This is consistent with need to replace buoyant parcel air when it moves upward. Equations of motion show that one can write the Laplacian of the pressure deviation in the parcel as the sum of a term associated with the vertical buoyancy gradient (F_B) and a dynamical term (F_D)

$$\nabla^2 p' = F_B + F_D \quad [14]$$

where $F_B = -\partial(\rho_0 B)/\partial z$ is the vertical gradient of buoyancy.

Ignoring the dynamic term, the buoyancy term is analogous to the role of charge density generating an electric field. A spatial arrangement of buoyancy will lead to a spatial distribution of pressure gradients $\nabla p'$. Force lines obtained from the solution of [14] and shown below, indicate the accelerations required to produce compensating motions in the environment needed to satisfy mass continuity when the buoyant parcel moves upward.

Vector field of buoyancy pressure-gradient force (BPGF) for a uniformly buoyant parcel of finite dimensions (shaded area). The positive and negative signs indicate the sign of the buoyancy forcing $-\partial(\rho_0 B)/\partial z$ along the top and bottom of the parcel.



Inside the parcel the lines of the BPGF force are against the buoyant acceleration. This is because, to move upward, the buoyant parcel needs to do work to move environmental air out of the way (to preserve continuity). Only way to avoid this is to have an infinitely narrow parcel. This illustrates that a given amount of buoyancy provides a larger upward acceleration the narrower the parcel.

The pressure gradient force in some large cumulus and cumulonimbus clouds can be comparable to the buoyant acceleration itself. The maximum magnitude achieved by the buoyancy term, is when the pressure gradient force exactly balances the buoyancy. This is the case of hydrostatic balance. Mathematically, this occurs when the horizontal extent of the buoyant perturbation is infinite.

A trade-off between the BPGF (which increases with cloud horizontal extent) and the entrainment (which becomes less important as the cloud horizontal extent grows and the thermal is “shielded” from entrainment effects – recall the $\Lambda \propto d^{-1}$ dependence on parcel size) can be used to argue that there will be an optimal cloud size that is most strongly accelerated upward. Too small and entrainment reduces the buoyancy; too large and the BPGF increasingly counters the buoyancy B.

Gust fronts

- Precipitation evaporation and the drag of the precipitation on the air both help to create conditions whereby strongly negatively buoyant air can form within the precipitating core of the cloud.
- This air accelerates downward and can spread out across the surface in what is termed a "cold pool" because the air is considerably colder than the surrounding environmental.
- The boundary between the cold and warm air is often referred to as a "gust front". The dynamics of gust fronts follow idealized density currents seen in the lab.

The temperature difference forms a density and therefore a pressure gradient. Integration of the horizontal momentum equation leads to a propagation speed *U* given by

$$U = \sqrt{2gh\frac{\Delta\rho}{\rho_0}} \approx \sqrt{2gh\frac{\Delta\theta}{\theta_0}}$$

Where g is acceleration due to gravity, h is the depth of the cold air, $\Delta\rho$ is the density difference and $\Delta\theta$ the potential temperature difference across the boundary between the cold and the warm air.

Cold pools occur in all cloud systems from marine stratocumulus to severe thunderstorms.

Contrasting cold pool/gust front properties for cold pools under marine stratocumulus with those associated with severe thunderstorms

Variable	Marine Sc	Severe thunderstorm
$\Delta\theta$	1°C	10°C
h	100-300 m	1-2 km
U	2-4 m s^{-1}	10-30 m s^{-1}