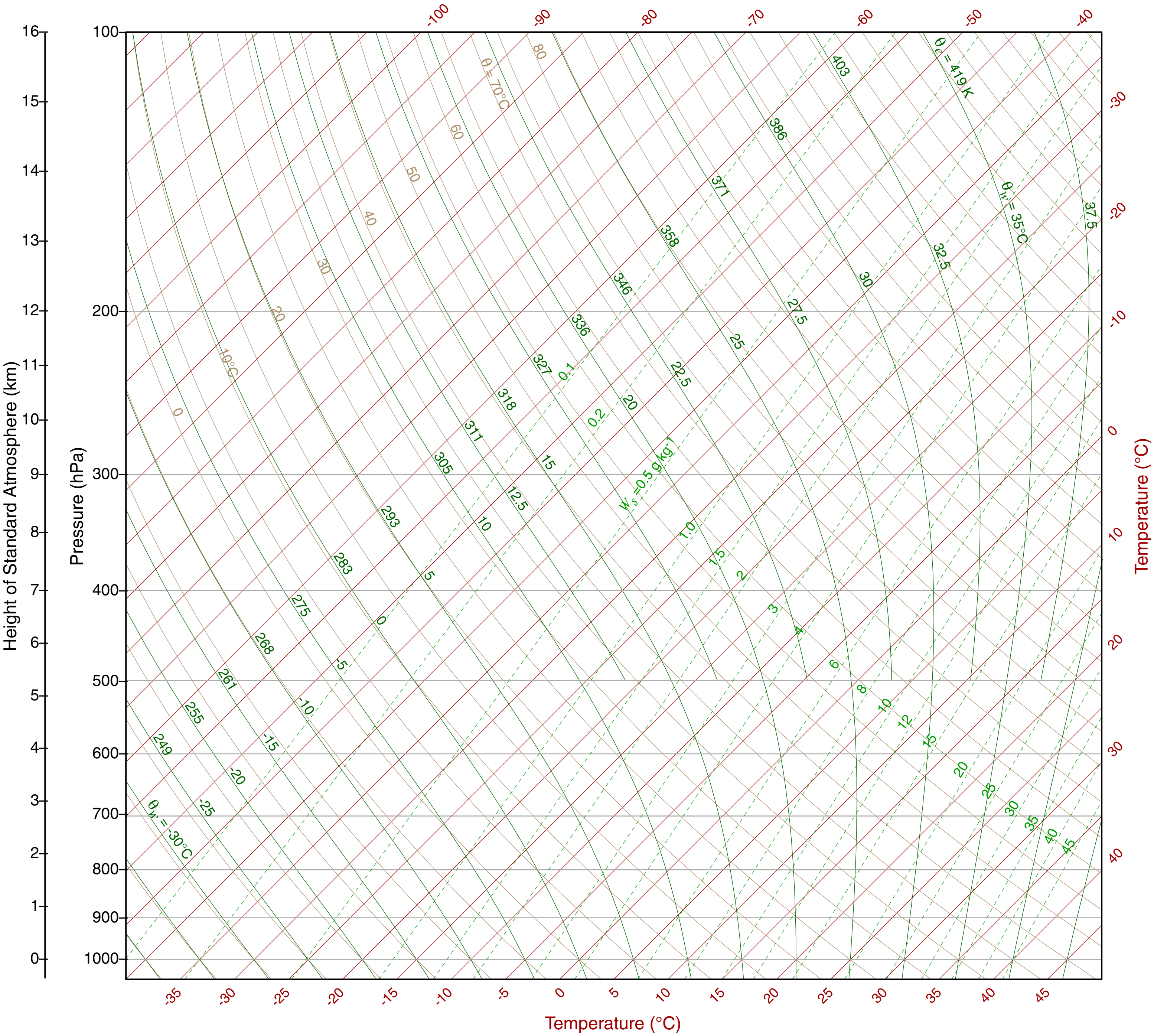
Skew T – ln p Chart



Courtesy of Jennifer Adams, COLA

Thermodynamic Chart Homework

Problem 1

Use a pseudo-adiabatic chart to fill in the following table.

p and other nonprimed variables (except LCL) refer to the initial condition of the parcel; primed variables refer to condition of parcel after lifting from *p* to *p′*.

p (mb)	T (°C)	T_d (°C)	θ (°K)	w (g/kg)	w_s (g/kg)	RH (%)	$T - T_d$ (°C)	LCL (mb)	p' (mb)	T' (°C)	T_d' (°C)	θ' (°K)	w' (g/kg)	w_s' (g/kg)	RH' (%)
950	5	0							800						
850			285			75			400						
920	-2			2.5					550						
1000	0						5		600						
1010	25							960	450						
890	12					65			450						
500	-19					50			1000						

See next page for answers

Thermodynamic Chart Homework

Problem 1
Answers

p (mb)	T (°C)	T_d (°C)	θ (K)	w (g/kg)	w_s (g/kg)	RH (%)	$T-T_d$ (°C)	LCL (mb)	p' (mb)	T' (°C)	$T_{d'}$ (°C)	θ' (K)	w' (g/kg)	w_s' (g/kg)	RH' (%)
950	5	0	282	4	5.8	69	5	880	800	-6	-6	285	3.0	3.0	100
850	-1	-4.7	285	3.15	4.2	75	3.7	810	400	-48	-48	293	0.13	0.13	100
920	-2	-6.8	277.5	2.5	3.6	69	4.8	855	550	-34	-34	283	0.4	0.4	100
1000	0	-5	273	2.63	3.8	69	5	930	600	-32.7	-32.7	279	0.42	0.42	100
1010	25	21.5	297	16	20	80	3.5	960	450	-10.5	-10.5	330	3.8	3.8	100
890	12	5.7	294.6	6.5	10	65	6.3	813	450	-26.5	-26.5	310	0.95	0.95	100
500	-19	-26.8	310	0.85	1.7	50	7.8	440	1000	37	-19.2	310	0.85	42	2

Atmospheric Sciences 535
Homework

Stability and Instability Criteria:

The criteria for buoyant stability and instability may be stated in terms of temperature lapse rates:

Absolutely unstable if $-\partial T_e / \partial z > \Gamma_d$

Conditionally unstable if $\Gamma_d > -\partial T_e / \partial z > \Gamma_m$

Absolutely stable if $-\partial T_e / \partial z < \Gamma_m$

Use an adiabatic chart to illustrate that these criteria may also be stated as:

Absolutely unstable if $\partial \theta / \partial z < 0$

Conditionally unstable if $\partial \theta_{es} / \partial z < 0$ and $\partial\theta/\partial Z > 0$

Absolutely stable if $\partial \theta_{es} / \partial z > 0$

Also use the adiabatic chart to illustrate that a layer is:

Potentially unstable if $\partial \theta_e / \partial z < 0$

Atmospheric Sciences 535
Homework
Sounding Evaluation

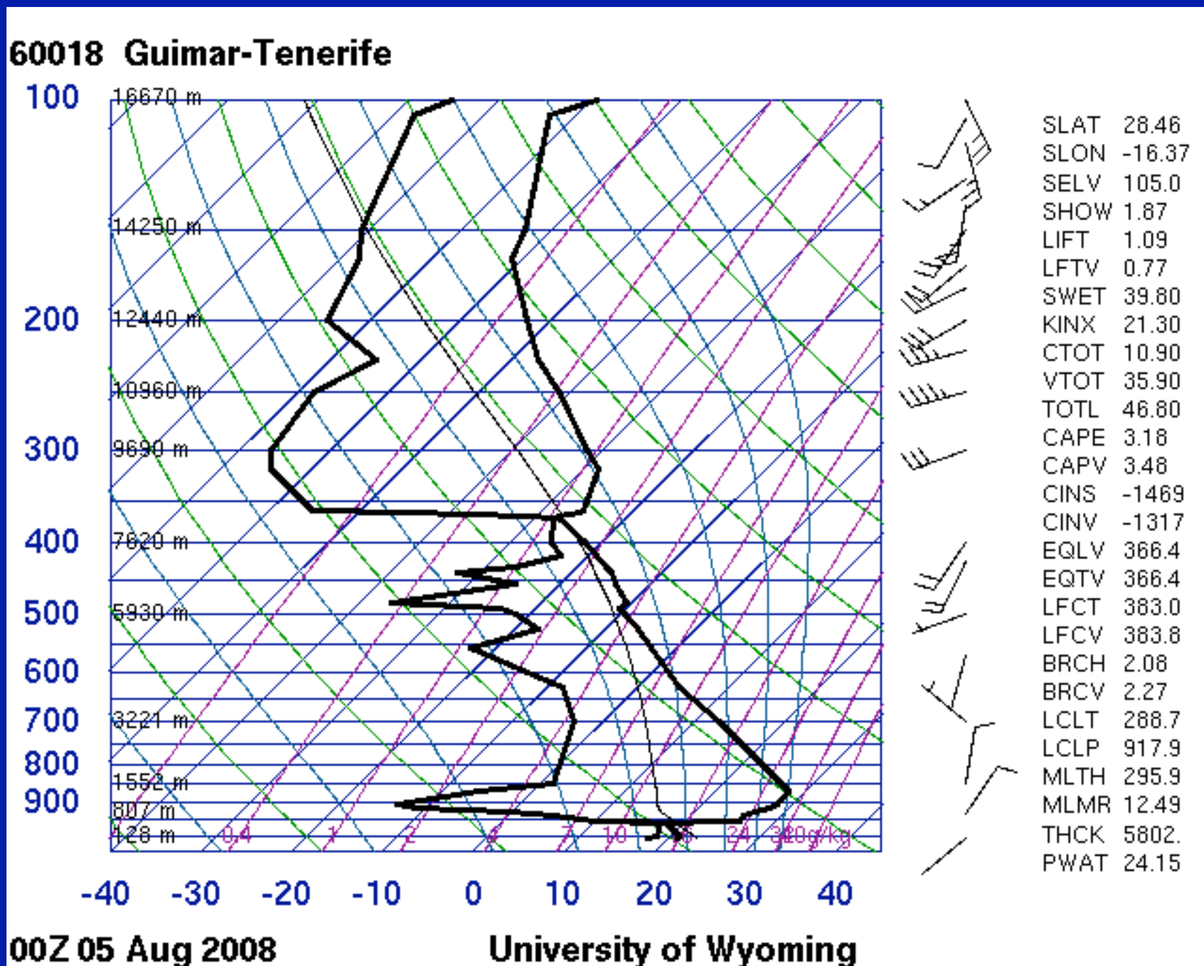
1. For each of the attached soundings, plot the values of θ, θ_e, and θ_{es} as functions of height. Also plot the relative humidity. You can download the data from http://weather.uwyo.edu/ .

2. Assess the stability of each layer of each sounding in terms of these variables.

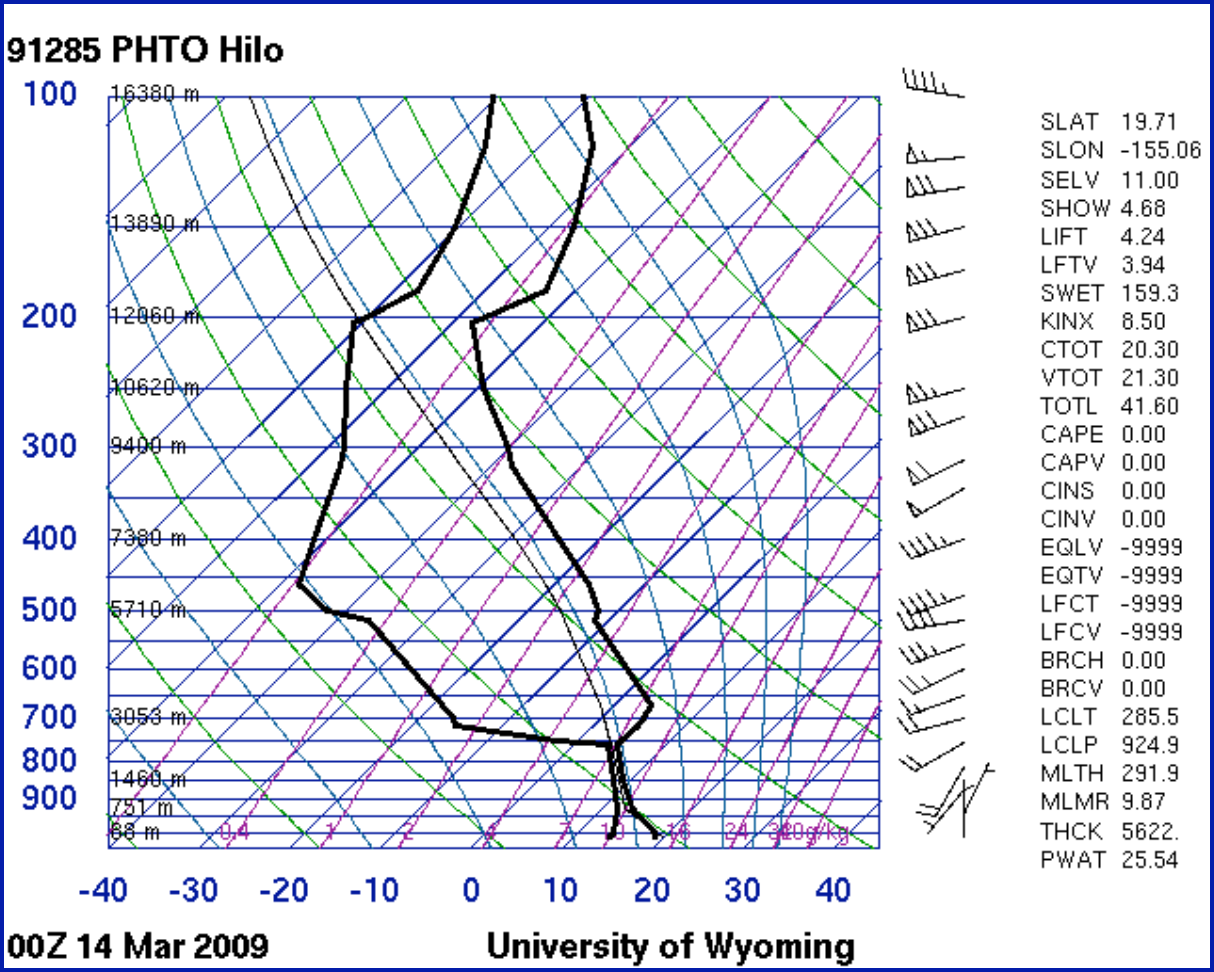
3. From these soundings, explain why deep convective clouds in the regions of Niamey are likely to have more lightning than those forming near Singapore.

4. Explain why the Oklahoma sounding is likely to support severe thunderstorms while the sounding at Hilo is only supportive of moderate trade cumuli. Feel free to invoke information based on the location of the sounding and the typically prevailing conditions in that location.

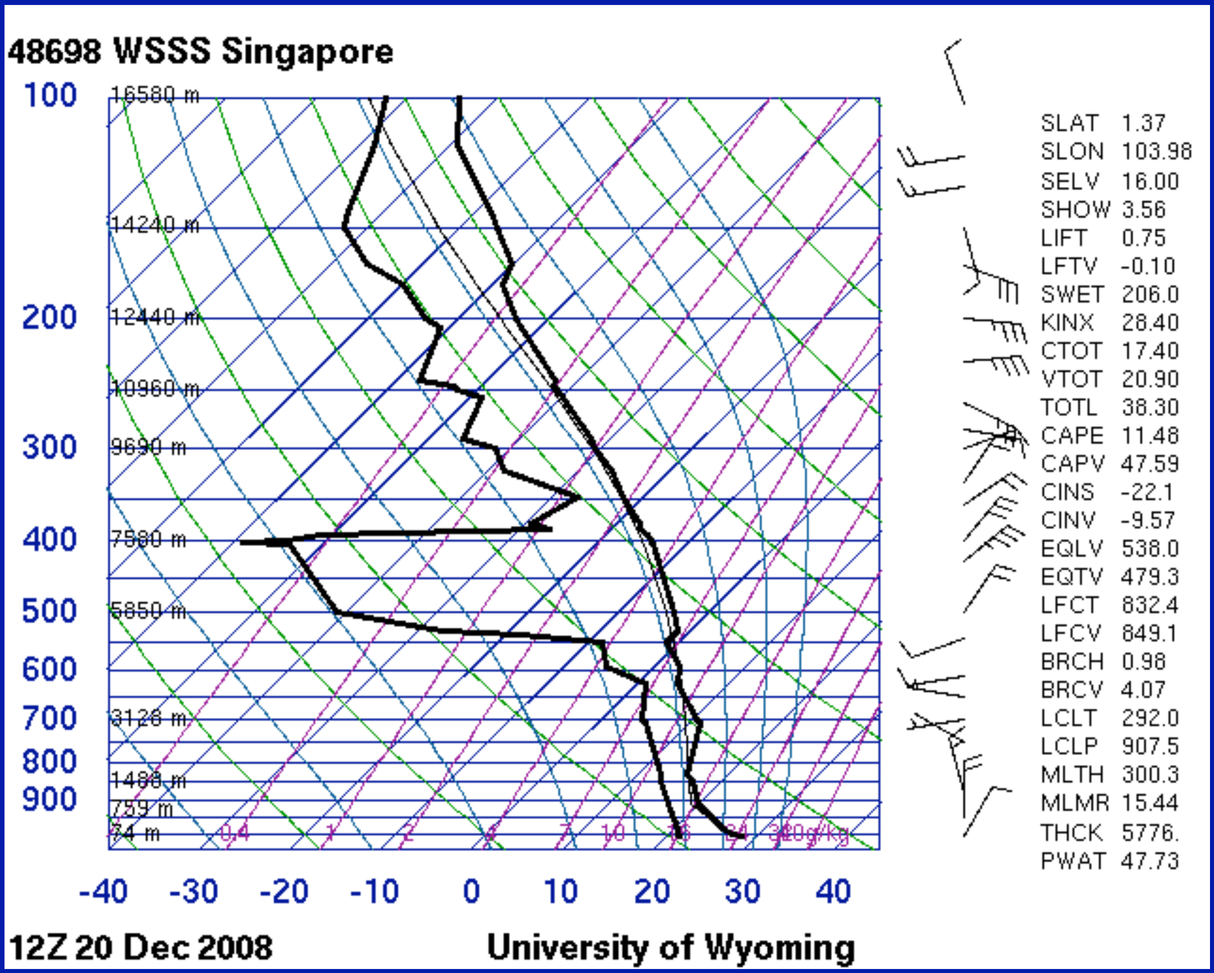
Stratocumulus Regime



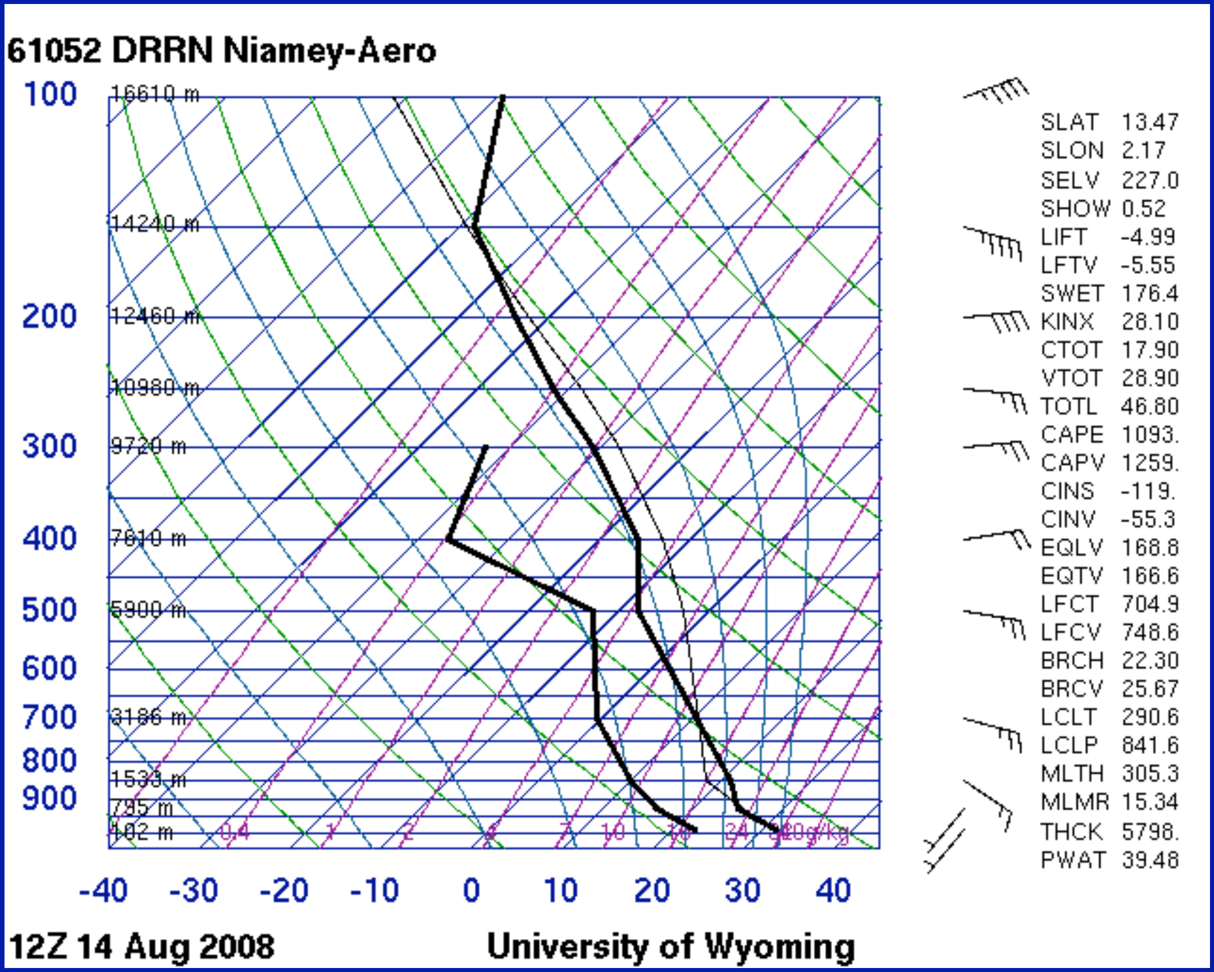
Trade Wind Regime



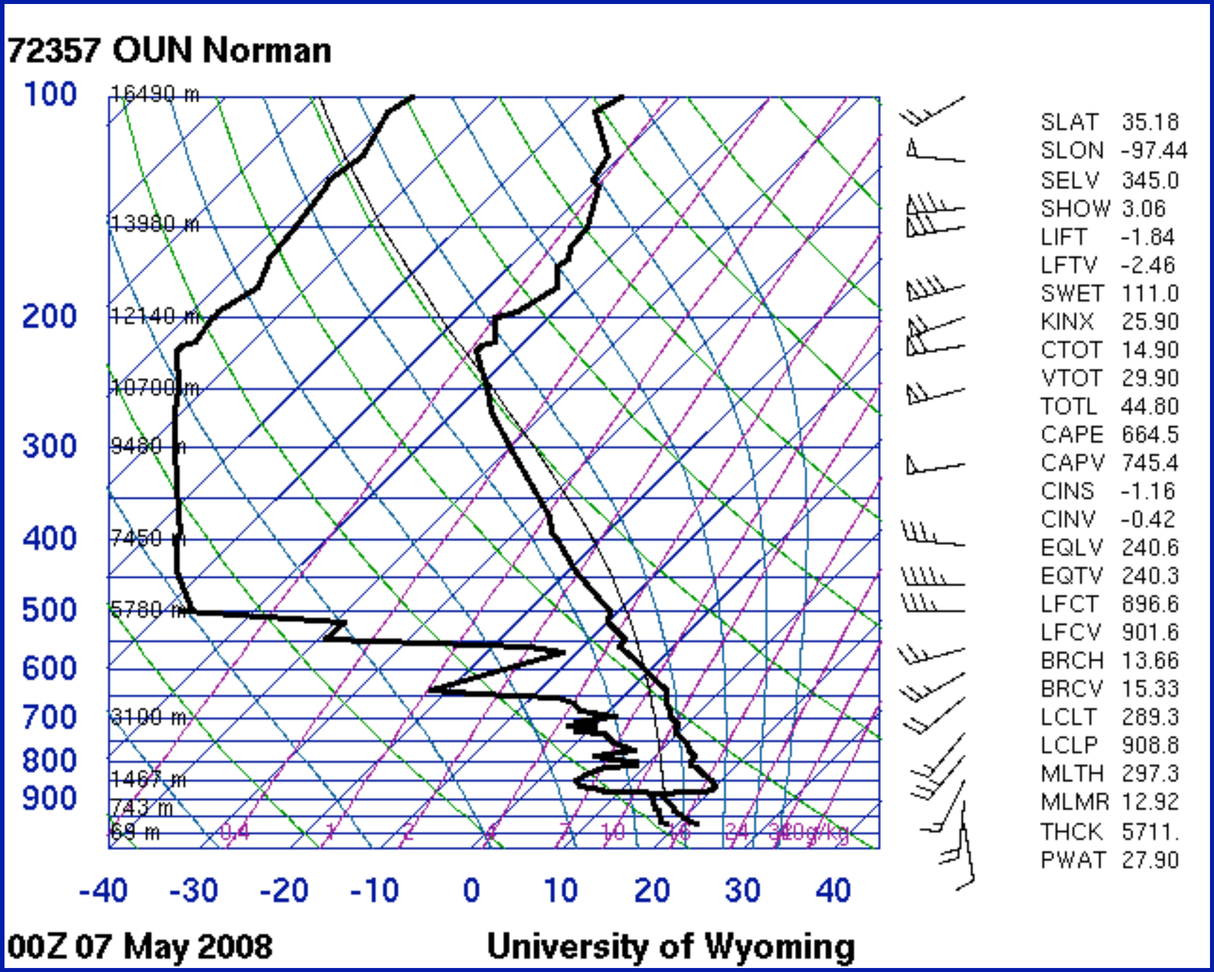
Indo/Pacific Warm Pool



West African Squall Line



Oklahoma Severe Thunderstorm



SKEW-T PROBLEMS

1) An air parcel containing cloudy air, initially at T = 0ºC and p = 800 hPa, is lifted to 700 hPa. It then mixes with an equal amount of ambient air that has T = -15ºC and a mixing ratio of 0.1 g kg^{-1}. Determine the temperature and mixing ratio of the new air parcel immediately after it has mixed (before any additional latent heating or cooling occurs). If enough water is now evaporated from the cloud droplets within the air parcel to again bring the air to saturation, what is the final temperature of the parcel? How much liquid water (expressed as kg liquid water per kg dry air) had to be evaporated?

2) An air parcel at p = 1000 hPa and T = 20ºC has a wet-bulb temperature of 15ºC. What is the dewpoint and mixing ratio of the air? If this air is lifted until all of the moisture condenses and falls out, and then the parcel is brought back down to 1000 hPa, what is the final temperature of the air?

3) An air parcel at 900 hPa has a temperature of 16ºC and a mixing ratio of 9 g kg^{-1}. What is the wet-bulb potential temperature of the air? The air is lifted to the 600 hPa level by passing over a mountain, and 75% of the water vapor that is condensed out by the ascent is removed by precipitation. Determine the temperature, potential temperature, mixing ratio, and wet-bulb temperature of the air parcel after it has descended to the 900 hPa level on the other side of the mountain.

4) An air parcel at 850 hPa has a temperature of 27.5ºC and a dewpoint of 10ºC. What is its mixing ratio? At what pressure is its LCL? What is its potential temperature? What is its equivalent potential temperature? What is its wet-bulb temperature? What is its wet-bulb potential temperature?

5) An air parcel at 650 hPa, with T = 7ºC and T_d = -22ºC, descends adiabatically to 900 hPa, and during its descent, rain falling through the parcel evaporates, so that w increases by 4 g kg^{-1}. What is the T of the parcel? What are its θ_w and θ_e?

6) Plot the following on a skew T – log p diagram and answer questions a through e (continued on reverse):

   Temperature follows the T = 300 K dry adiabat from 1000 to 900 hPa; then follows a straight line to the point p = 830 hPa, T = 20ºC; then to the point p = 550 hPa, T = -10ºC; then to the point p = 250 hPa, T = -55ºC; then isothermal above that point. Dewpoint follows the 12 g kg-1 saturation mixing ratio line from 1000 to 900 hPa; then follows a straight line to the point p = 830 hPa, T = -2ºC; then to the point p = 150 hPa, T = -100ºC.

a) For each of the following layers, characterize the lapse rate as either absolutely unstable, dry neutral (or dry adiabatic), conditionally unstable, moist neutral (or moist adiabatic), or absolutely stable. Also indicate if a layer is an inversion.
   1000-900 hPa
   900-830 hPa
   830-550 hPa
   550-475 hPa
   475-400 hPa
   400-250 hPa
   Above the 250 hPa

b) Prove that the layer from 900 to 830 hPa is potentially unstable.

c) Now draw the temperature profile of a surface air parcel lifted up to 150 hPa. What is the LCL of the parcel? What is the LFC of the parcel? What is the EL of the parcel?

d) Indicate with hatching the CIN and CAPE for this parcel on the diagram.

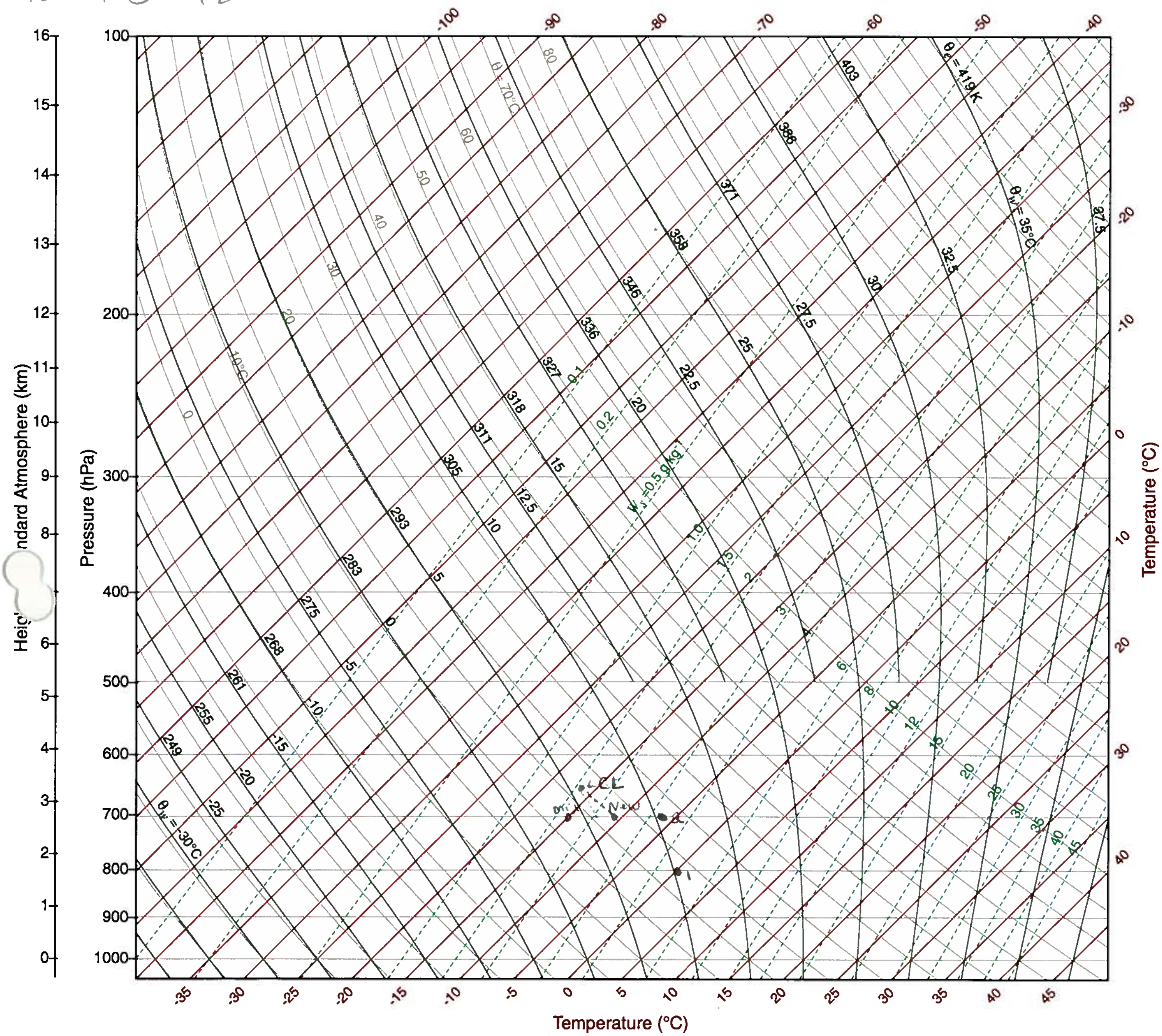
e) Estimate CAPE. Ignore T_v effect. Assume that the reference air parcel in the layer between the LFC and EL is on average 5ºC warmer than environmental air at the same level.

$T_1 = 0°C$
1) $P_1 = 800 hPa$
$P_2 = 700 hPa$
$w_1 = 4.5 g kg^{-1} = w_1$
$w_2 = 3.2 g kg^{-1} = w_2$
$T_2 = -7°C = T_2$

$T_{mix} = -15°C$
$w_{mix} = 0.1 g kg^{-1}$
$P_{mix} = 700 hPa$

$T_{new} = (-7 + -15°C)/2 = -11°C$
$w_{new} = (3.2 + 0.1 g kg^{-1})/2 = 1.65 g kg^{-1}$
$w_s = 2.2 g kg^{-1}$

Skew T – ln p Chart



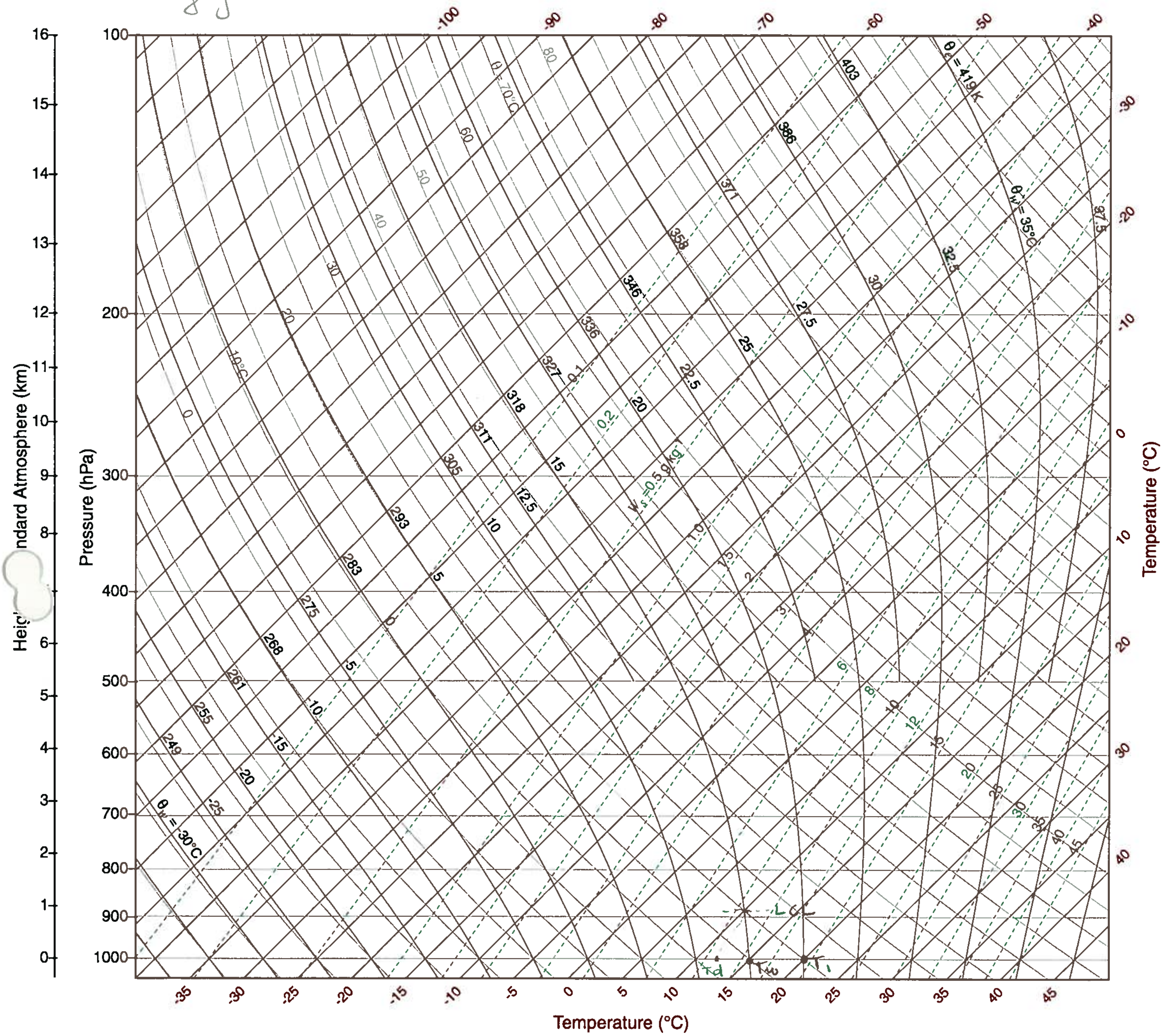
Courtesy of Jennifer Adams, COLA

$w_{evap} = 2.2 - 1.65 = 0.5 g kg^{-1}$

after evap $T = T_w = -12.5°C$

2:) p_i = 1000 hPa
T_1 = 20°C
T_w = 15°C
T_d = 12°C
w = 8.5 g kg⁻¹

Skew *T* – ln *p* Chart



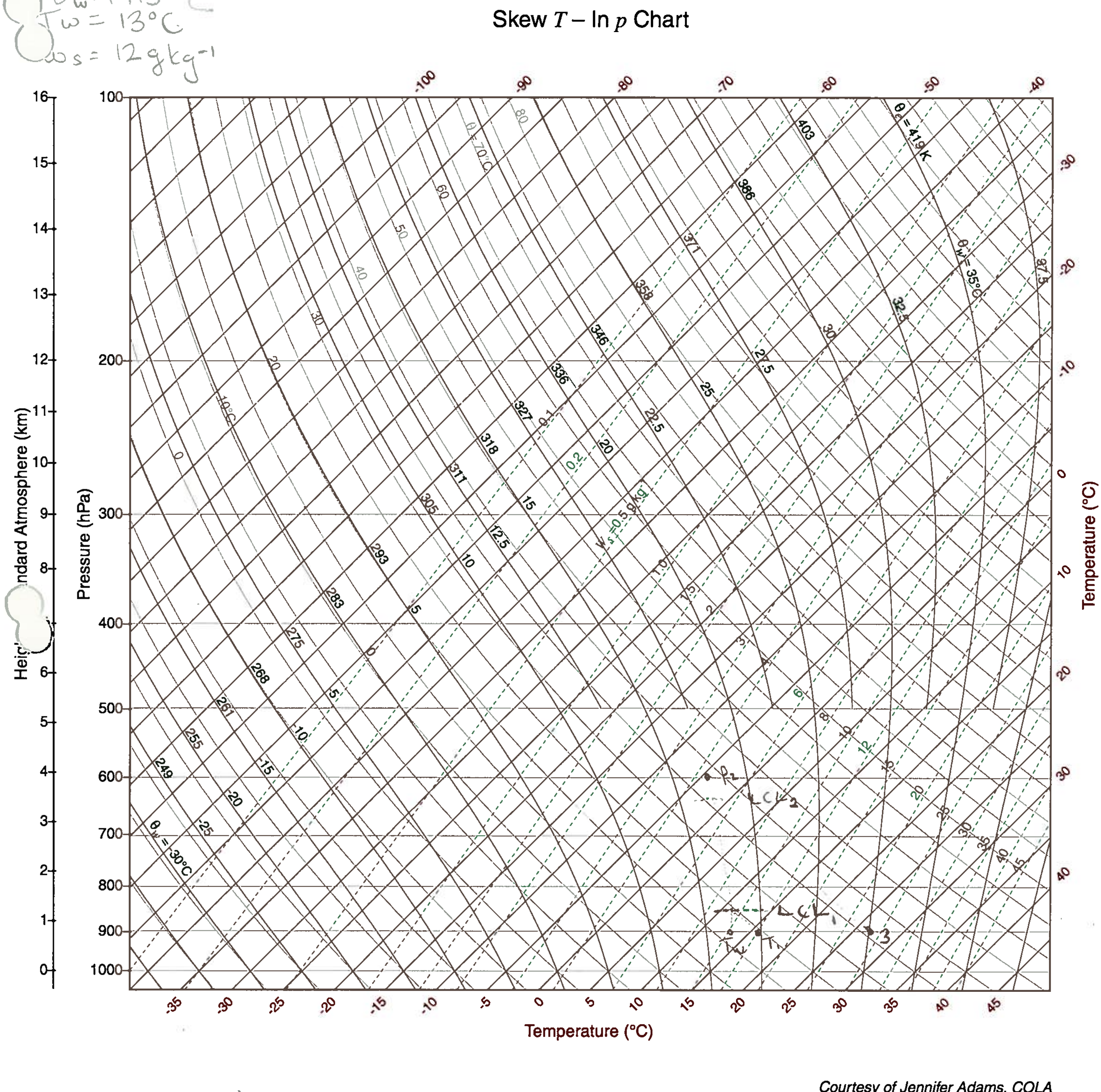
Courtesy of Jennifer Adams, COLA

$T_w = \theta_e$ @ 1000 hPa = 318 K

Def. of eq pot T (θ_e) is T an air parcel would have if all moisture condensed out and it was lowered adiabatically to 1000 hPa.

3.) $p_1 = 900$ hPa
$T_1 = 16°C$
$w_1 = 9$ g kg^{-1}
$\theta_w = 17.5°C$
$T_w = 13°C$
$w_s = 12$ g kg^{-1}

$p_2 = 600$ hPa
75% of condensed water precipitates
$p_3 = 900$ hPa



$w_2 = 4.5$ g kg^{-1}

$w_1 - w_2 = 9 - 4.5 = 4.5$ g kg^{-1} condensed

$0.75 \times 4.5 = 3.4$ g kg^{-1} precipitates, 1.1 g kg^{-1} remains condensed

$w_3 = 4.5$ g kg^{-1} + 1.1 g kg^{-1} = 5.6 g kg^{-1}

$T_3 = 27°C$ $\quad T_{w_3} = 13°C$

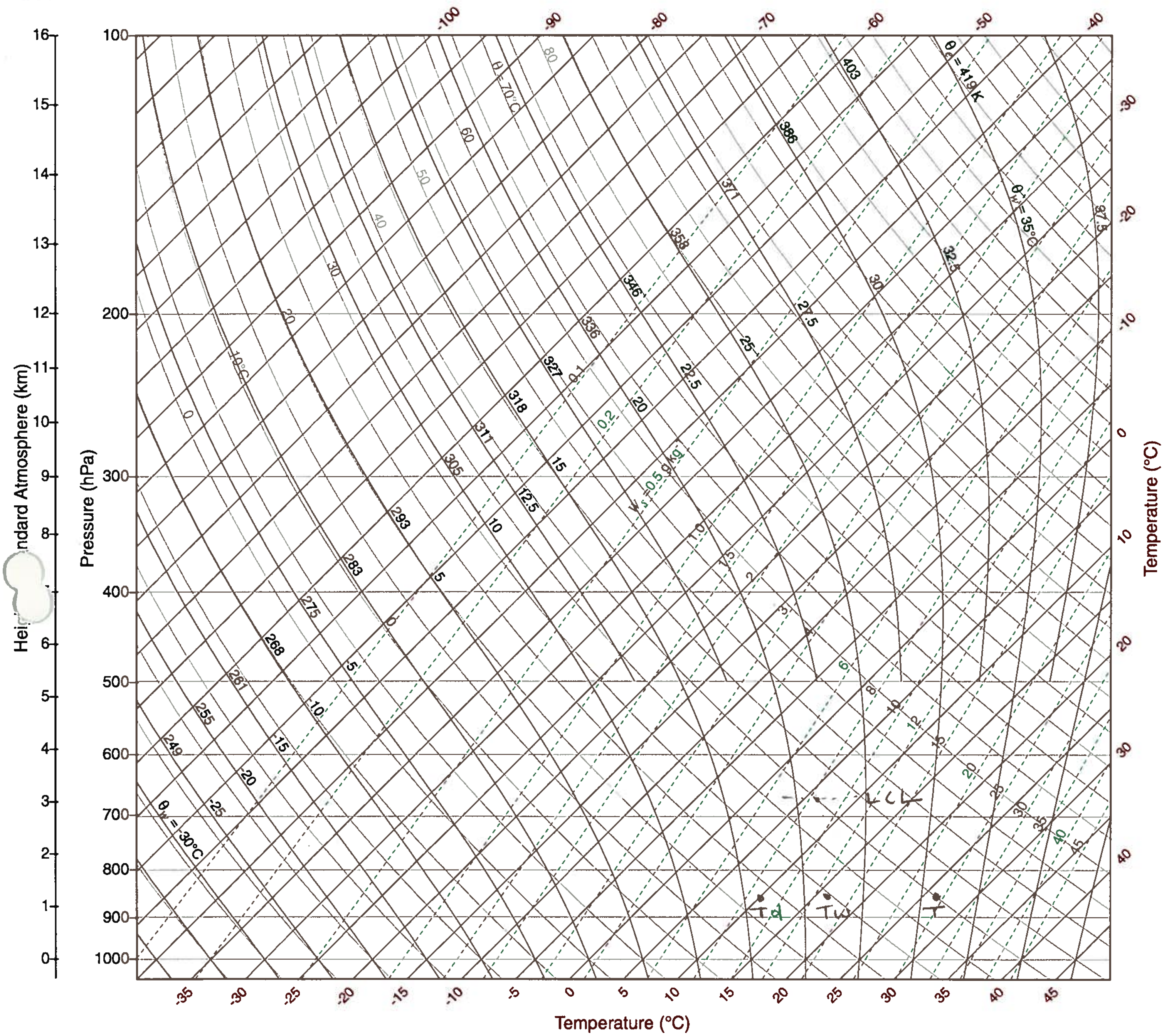
$\theta_3 = 37°C$

4) $p_i = 850$ hPa

$T_1 = 27.5$

$T_d = 10°C$

Skew T – ln p Chart



Courtesy of Jennifer Adams, COLA

$W = 8.5$ g kg^{-1}

LCL = 670 hPa

= 40°C = θ

= 341 K = θ_e

T_w = 17°C

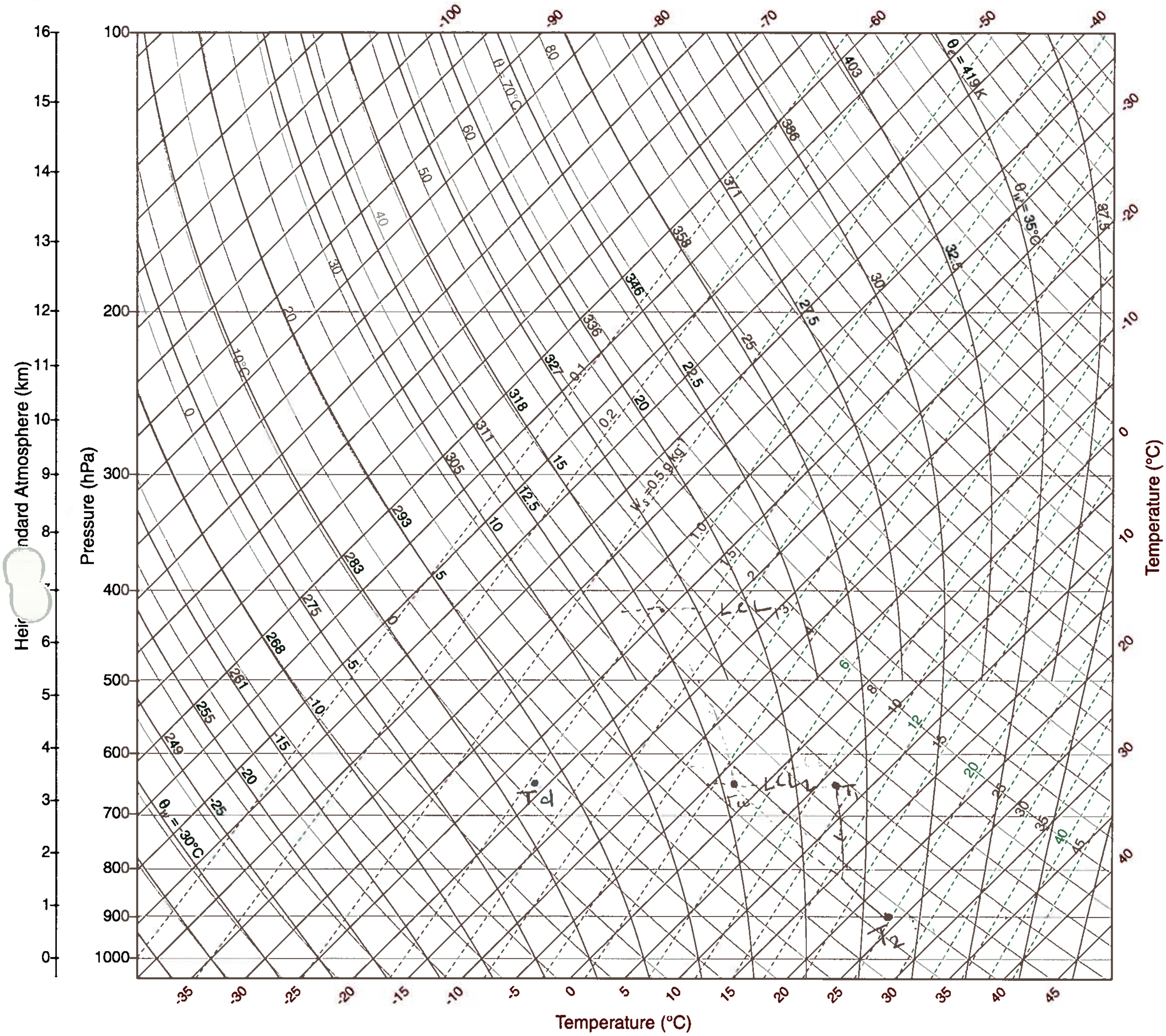
θ_w = 21°C

5.) $p_1 = 650$ hPa

$T_1 = 7°C$

$T_d = -22°C$

Skew T – ln p Chart



Courtesy of Jennifer Adams, COLA

$w_1 = 1.1 \text{ g kg}^{-1}$

$w_2 = 1.1 + 4 = 5.1 \text{ g kg}^{-1}$

$T_2 = 23°C$ ⟶ follow, from T_1, moist adiabat down 4 g kg^{-1}, then dry adiabat to $p = 900$ hPa

$\theta_w = 16°C$

$\theta_e = 322$ K

c) LCL = 860 hPa
LFC = 675 hPa
EL = 222 hPa

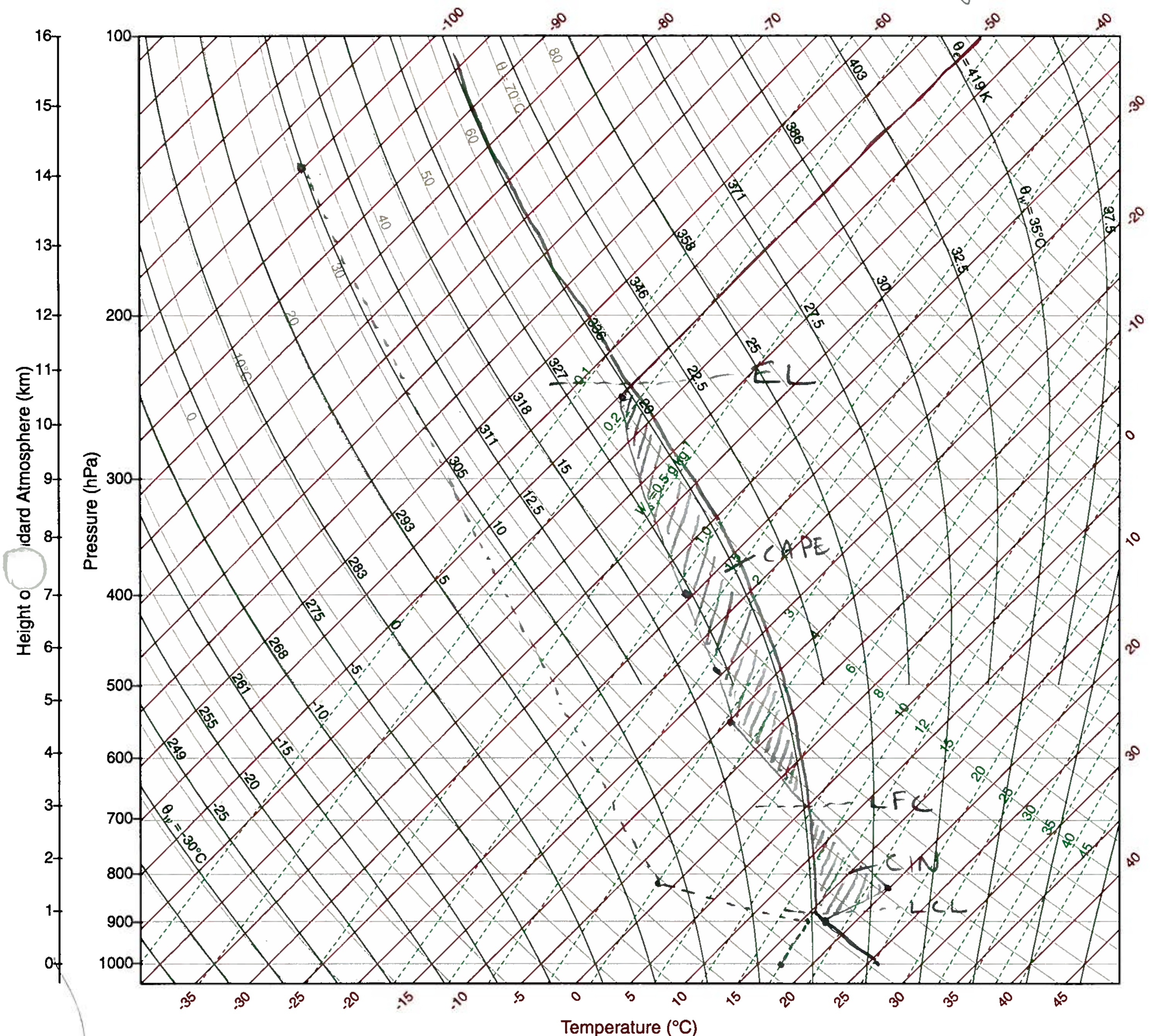
e) $CAPE = \int_{LFC}^{EL} \frac{T_p - T_e}{T_e} g\,dz = R \int_{EL}^{LFC} (T_p - T_e)\, d(\ln p)$

$p_1 = LFC = 675\ hPa$
$p_2 = EL = 222\ hPa$

$= (287)(5\ ^\circ C) \ln\left(\frac{675}{222}\right)$

$= 1595\ J\,Kg^{-1}$

Skew T – ln p Chart



Courtesy of Jennifer Adams, COLA

a) 1000-900 hPa — dry neutral — $\Gamma = \Gamma_d$
900-830 hPa — absolutely stable — $\Gamma < \Gamma_m$
830-550 hPa — conditionally unstable — $\Gamma_m < \Gamma < \Gamma_d$
550-475 hPa — " " "
475-400 hPa — moist neutral — $\Gamma = \Gamma_m$
400-250 hPa — absolutely stable — $\Gamma < \Gamma_m$
above 250 hPa — " " "

b) $\theta_e @ 900 = 338K$
$\theta_e @ 830 = 322K$

$\frac{d\theta_e}{dz} < 0$ ∴ convectively unstable

$$CIN = R \int_{LFC}^{\cancel{LCL}\ 1000\,mb} (T_p - T_e)\, d(\ln p)$$

$$= 287\,(5^\circ C)\ \ln\left(\frac{\cancel{860}\ 1000}{675}\right)$$

$$= \cancel{347}\ 564\ J\,kg^{-1}$$

Homework – Thermodynamics and Cloud Vertical Structure

1. Derive an expression for the rate of increase of relative humidity with respect to height for a well-mixed unsaturated layer. Use this expression to evaluate the depth of layer required so that the top is just saturated given that the base has a relative humidity RH of 30%. What does this tell you about the likely outcome of mixing in the troposphere (where RH is typically 30% or more)?

2. (a) Using thermodynamic theory, derive an expression for the rate dq_L/dz with which liquid water mixing ratio increases with height in an adiabatic cloud (i.e. a cloud in which the temperature lapse rate is moist adiabatic and the total water mixing ratio is conserved). Tip: start with an expression for the moist adiabatic lapse rate.

(b) Upon which fundamental properties of air does this rate most strongly depend? Examine this by calculating dq_L/dz, using a simple parameterization (e.g. Teten's formula) for the saturation vapor pressure, for the four combinations of two temperatures and pressures (e.g. T=270 K and T=290 K, p=700 hPa and p=900 hPa).

(c) Estimate the vertically integrated liquid water content (known as the liquid water path, or LWP, the integral of ρq_L over height, where r is the air density) for a cloud layer of depth 2 km. You can assume that the cloud is at a constant pressure and temperature of 900 hPa and 290 K. Compare this value with microwave estimates of cloud LWP from satellite sensors:

http://www.remss.com/ssmi/ssmi_browse.html

What conclusion(s) can you draw (there are several possibilities) about the nature of clouds structure?

(d) Suggest a way in which your expression could be used to infer aspects of cloud feedbacks in response to a warming climate.

3. (a) Using the concepts by which one derives an atmospheric scale height, try to come up with an expression for the scale height of moisture by incorporation of the Clausius-Clapeyron equation. Assume that the relative humidity is constant with height.

(b) How does this scale height vary with temperature and how does it compare with the atmospheric scale height?

(c) What does your result tell you about the height that a mountain range would need to have so that all of the moisture is rained out when an airmass passes over it?