A BIOMECHANICAL MODEL OF THE FOOT

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Abstract—The foot is modeled as a statically indeterminate structure supporting its load at the heads of the five metatarsals and the tuberosity of the calcaneous. The distribution of support is determined through an analysis of the deformations caused in the structure as a result of the forces at these locations. The analysis includes the effect of the plantar aponeurosis and takes into account the deformation of the metatarsals and bending of the joints. A parametric study is presented to illustrate the behavior of the solution under a broad range of conditions.

INTRODUCTION

Knowledge of the stresses that occur throughout normal and abnormal feet under various loading conditions is of fundamental importance in understanding the physiology and pathophysiology of the foot, and can provide important information for the prevention and treatment of injuries. The foot is an arch shaped structure carrying loads applied to the talus by the tibia and fibula. In an innovative early study, Hicks (1955) recognized that the foot maintained its arched shape under loading because of its ability to act as both a truss and a beam. An arch shaped structure supported at its ends acts as a truss when the ends are prevented from moving apart. In a bridge this is accomplished by an embankment at either end, or by a tie between the pieces. In the foot this tie is provided by the plantar aponeurosis, which extends from the tuberosity of the calcaneus to the phalanges just distal to the metatarsophalangeal joints. Tension in the plantar aponeurosis, necessary for the existence of truss action, was demonstrated by Hicks (1955). The tension is increased when the toes are extended upward at the metatarsophalangeal joints, and the foot is pulled into a high arch position by the plantar aponeurosis in a windlass mechanism, as described by Hicks (1955). This truss action subjects the metatarsals to compressive stress. When the ends can move apart, an arch supported at its ends can act as a beam if it is sufficiently rigid. Although the foot is composed of many bones, these bones are held together by ligaments, and the foot can maintain its arch while supporting load, thus acting as a beam and submitting the metatarsals to bending stress. Hicks (1954) suggested that the interplay between truss action, dominant in the heel up position when the windlass effect increases tension in the plantar aponeurosis, and beam action, significant when the foot is flat and the toes are straight, results in large variation of bending stresses on the metatarsals. He supported this concept through a simple but clever experiment, in which he mounted mirrors at the base and head of each metatarsal and measured the amount of bending by determining the angle of deflection of reflected light. The functional significance of the arch in the mechanics of the foot has also been discussed by Zitzlsperger (1958, 1960).

The major support areas of the foot are concentrated under the heads of the five metatarsals and the tuberosity of the calcaneus. Experiments designed to determine the distribution of loading among these six areas have employed a variety of techniques, ranging from such primitive early ideas as measuring indentations in sand or in bags filled with plaster, to the use of complex arrangements of a multitude of pressure transducers that provide data for the entire field of the foot-floor contact area. The wide range of results obtained from such experiments, not only among different researchers but also within individual studies, underscores the need for a better understanding of the factors that influence the manner in which the foot bears its load. The analysis presented here describes how the support is distributed among the metatarsal heads, and how this distribution depends on the orientation of the foot and on its physiological, anatomical and biomechanical properties.

A review of some of the early studies of foot-ground pressure distribution was given by Elftman (1934), who devised an improved method in which a subject stood on a rubber mat that contained pyramidal projections.
This was placed on a heavy glass plate, so that the flattening of the projections, indicating the magnitude of the load, could be observed from below. Draganich et al. (1980) described some of the initial uses of electronic devices for determining foot-floor contact pressures, and introduced an improvement by employing a system of pressure transducers consisting of a matrix of 7168 switches that provide a detailed picture of foot-floor contact pressure. Cavanagh and Ae (1980) introduced a visually effective technique for displaying pressure distribution measurements, and Aritomi et al. (1983) used thin pressure-detecting sheets to correlate pressure distribution with X-ray examination of the foot. The clinical significance of determining load distribution under the foot was emphasized in the work of Dhanendran et al. (1980), Hutton and Dhanendran (1981), Schneider and Chao (1983) and Katoh et al. (1983). Stokes et al. (1979) measured pressure distribution under the foot and then estimated the loads on the metatarsophalangeal joints and the metatarsal bones.

Because the foot is supported at six locations, it forms a statically indeterminate structure, and the manner in which this support is distributed can be determined only through an analysis of the deformations caused in the structure as a result of the forces at these locations. Our analysis of the deformation of the foot under applied loads assumes that the calcaneus, talus, navicular, cuboid and cuneiforms are rigid, that the metatarsals are flexible, and that all joints except the talocalcaneal joint are flexible. Deformation of the metatarsals is determined using beam theory. This initial study is restricted to the case where all muscles of the foot are relaxed, so that only the intersegmental ligaments limit relative movement of the bones. In some circumstances the toes, particularly the hallux, bear some of the load; this possibility will not be included in the present analysis. However, the role of the tendocalcaneus and the tibialis anterior will be included, so that the effect of shifting the load anteriorly or posteriorly can be studied. The plantar aponeurosis is of fundamental importance for the biomechanics of the foot, and its effect will be included. The stress-strain relationships for ligaments show a strong non-linear effect for very small loading, followed by a linear portion that applies over the range of normal physiological loading (see, for example, Woo et al., 1982). In the present analysis, it will be assumed that except for very small applied forces, deformation of the plantar aponeurosis and of the ligaments joining the bones of the foot can be related linearly to the applied load.

**ANÁLISIS**

Small amounts of force applied to the unloaded foot cause it to take on a wide variety of orientations. To define a unique initial state, we assume that all of the metatarsal heads lie on a horizontal plane, and that the heel is either on this plane or raised to a specified level above it. The toes are not bent at the metatarsophalangeal joints, so the windlass effect does not induce a stress on the plantar aponeurosis. In this initial state a small load is applied to the foot to take up the slack in the joints, and to stretch the ligaments and plantar aponeurosis into the linear portion of their stress-strain relationships. This load is small compared to the normal load that the foot must support, and so its magnitude may be neglected. The foot is then loaded by keeping the talus in a fixed position and raising the plane on which the metatarsal heads lie, while simultaneously rotating the toes into a horizontal position.

To maintain the heel in a raised position, a moment must be provided by the tendocalcaneus. This moment changes as the heel drops and becomes level with the floor. Relaxation of the gastrocnemius occurs as the portion of the total load that is borne by the heel increases. Additional shifting of the load onto the heel may continue, even after total relaxation of the gastrocnemius, since a moment of the opposite sign can be provided by the tibialis anterior. We will specify the fraction of the total load that is supported by the heel and determine the moment, either positive or negative, that must exist to achieve this.

The fraction of the load not supported by the tuberosity of the calcaneus is borne by the heads of the five metatarsals. The loading must be distributed among the metatarsals in such a way that the metatarsal heads remain coplanar in the deformed state. The load distribution is therefore related to the relative rigidity of each of the five rays. We assume that the rays deflect as a result of deformation of the metatarsals and bending at each of the joints.

Figure 1 is a dorsal view, illustrating the bones in the foot. Figure 2 is a cross-section showing one ray. Since there are two bones shown in the midfoot, this figure corresponds to one of the first three rays. A coordinate system is chosen with the $x$-axis tangent to the tuberosity of the calcaneus and passing through the point where the metatarsal head meets the horizontal plane. The $y$-axis passes through the point of application of the load on the talus, and is perpendicular to the $x$-axis. Each ray has a different length, and the metatarsal heads touch the horizontal plane at different locations. Therefore the coordinate system is different for each ray, and will be denoted by $(x_j, y_j)$, $j = 1−5$. The $x_j$-axis makes an angle $\theta_j$ with the horizon-
A biomechanical model of the foot

Fig. 2. A cross-section through one ray, illustrating the coordinate system.

tal, and the heel is elevated an amount $h > 0$ above the horizontal.

The external forces acting on the foot are illustrated in the figure. Vertical forces $R_j$, $j = 1-5$, act on the head of each metatarsal, and a vertical force $R_p$, assumed non-zero only when the heel is down ($h = 0$), acts on the tuberosity of the calcaneus. The load on the foot, $W$, is applied to the talus. We assume all forces are normalized by $W$, and all lengths by $L$, a characteristic length of the foot. Therefore, the load on the talus is shown in Fig. 2 as a downward force of magnitude 1.

There is also a moment $M$, assumed positive clockwise, that is provided either by the tendocalcaneus or the tibialis anterior. A balance of these forces and moments gives

$$\sum_{j=1}^{5} R_j = 1$$

and either

$$\sum_{j=1}^{5} d_j R_j = M$$

when the heel is up, or

$$\sum_{j=1}^{5} d_j R_j + x_R R_0 = M$$

when the heel is down ($h = 0, \theta_j = 0, j = 1-5$). Clearly, $d_j = x_{ij} \cos \theta_j - y_{ij} \sin \theta_j$, where $x_{ij}$ is the $x_j$ coordinate of each metatarsal head and $y_{ij}$ is the $y_j$ coordinate of the point of application of the load. Also, $x_R$, the moment arm for $R_p$, is independent of $j$ and uniquely defined when $h = 0$.

Additional equations for the unknown forces are obtained by requiring that after deflection the metatarsal heads remain coplanar. The deflections are due not only to the applied loads $R_j$, but also to the forces $T_j$ induced by the plantar aponeurosis, directed along the negative $x_j$-axes.

Let $u_j$ and $v_j$ denote the $x_j$ and $y_j$ displacements, respectively, of the metatarsal heads under the action of $R_j$ and $T_j$, $j = 1-5$. The vertical component of this displacement is $v_j \cos \theta_j - u_j \sin \theta_j$. We can account for pronation and supination by specifying that after deformation the metatarsal heads lie on a plane that is slanted in the $z$-direction relative to the horizontal, where the $z$-axis is perpendicular to $(x_j, y_j)$. If $z_j$ is the $z$ coordinate of each metatarsal head, $j = 1-5$, and $\Delta_j = z_j \tan \beta$, where $\beta$ is the angle of inclination of the plane along which the metatarsal heads lie after deformation, it follows (see Fig. 3) that

$$v_1 \cos \theta_1 - u_1 \sin \theta_1 - \Delta_1 = \ldots$$

$$- v_5 \cos \theta_5 - u_5 \sin \theta_5 - \Delta_5.$$ (3)

Since $u_j$ and $v_j$ will be determined in terms of $T_j$ and $R_p$, these represent four additional equations for the unknown forces.

The force $T_j$ exerted by the plantar aponeurosis on each ray is a function of the displacement $u_j$ in the $x_j$ direction. We assume that the small initial loading discussed earlier has stretched the plantar aponeurosis through the non-linear region, and that a linear constitutive relationship may be used. Stress is induced in the plantar aponeurosis not only through deflections $u_j$ of the metatarsal heads, but also through the windlass effect induced by bending of the toes through the angles $\theta_j$ (Fig. 4). Therefore,

$$T_j = k_j u_j + v_j \theta_p$$

for suitable constants $k_j$ and $v_j$.

Fig. 3. After deformation the metatarsal heads lie on a plane inclined at angle $\beta$ to the horizontal.

Fig. 4. The windlass effect: when the heel is lifted tension in the plantar aponeurosis is increased.
We have derived eleven equations for the eleven unknowns \( M, R_j, T_j, j = 1-5 \). To complete these equations it is necessary to determine the displacements \( u_j, v_j \) in terms of \( K_r \) and \( I_j \). These displacements are a result of the deformation of each of the metatarsals, and the deflections of each of the joints of the foot.

**THE DEFORMATION OF THE METATARSALS**

The metatarsals are curved beams of variable cross-section, with center-lines described by \( x_j(s), y_j(s), 0 \leq s \leq l_j, \) where \( l_j, j = 1-5, \) are the bone lengths. We determine the deflection of the metatarsal heads under the action of forces \( U_j \) and \( V_j \) in the negative \( x_j \) and positive \( y_j \) directions, respectively, where

\[
\begin{align*}
U_j &= R_j \sin \theta_j + T_j, \\
V_j &= R_j \cos \theta_j, \quad j = 1-5.
\end{align*}
\]

(5)

At any location \( s \) along a metatarsal, the internal forces on a cross-section perpendicular to the center-line are equivalent to a resultant force through the centroid with normal component \( N_j(s) \), and a couple of moment \( M_j(s) \), given by

\[
\begin{align*}
N_j(s) &= V_j \sin \phi_j(s) + U_j \cos \phi_j(s) \\
M_j(s) &= \left( x_{oj} - x_j(s) \right) V_j - y_j(s) U_j
\end{align*}
\]

(6)

where \( (x_{oj}, 0) \) are the coordinates of the metatarsal head \( (x_{oj} = x_j(l_j)) \). The angle \( \phi_j \) between the tangent to the center-line and the horizontal, shown in Fig. 5, satisfies \( 0 \leq \phi_j \leq \pi/2 \). If \( d\phi \) is the difference in \( \phi \) at the locations \( s \) and \( s + ds \), the moment \( M_j \) decreases this difference by \( \Delta(d\phi) = M_j(s) ds/El_j(s) \), while the normal force \( N_j \) decreases the length \( ds \) by \( \Delta(ds) = N_j(s) ds/EA_j(s) \). Here \( I_j(s) \) and \( A_j(s) \) are the normalized cross-sectional moment of inertia and area as a function of distance \( s \) along the metatarsal, and \( E = E_0 L_j^2/W_j \), where \( E_0 \) is Young's modulus for bone.

The horizontal and vertical displacements of the metatarsal head due to the changes in \( d\phi \) and \( ds \) at \( x_j(s), y_j(s), \) are, respectively

\[
\begin{align*}
d \bar{u}_j &= \Delta(d\phi) t_j(s) \sin \psi_j(s) - \Delta(ds) \cos \phi_j(s) \\
d \bar{v}_j &= \Delta(d\phi) t_j(s) \cos \psi_j(s) + \Delta(ds) \sin \phi_j(s)
\end{align*}
\]

(7)

**DEFLECTIONS DUE TO BENDING OF JOINTS**

We now determine the deflections of the metatarsal heads under the applied forces due to passive bending of each of the joints as a result of elongation of the supporting ligaments. Only the talus and calcaneus will be regarded as rigidly joined, an assumption justified by their large articulating surface and numerous supporting talocalcaneal ligaments. Each of the other joints will be assumed to deflect through an angle proportional to the moment acting on it. To determine these moments, we denote by \( (x_{oj}, y_{oj}), j = 1-5, \) the location of the point of rotation of each of the joints. For the rays \( j = 1, 2, 3 \), there are three joints, the metatarsalcuneiform, cuneonavicular and talonavicul, corresponding to \( n = 1, 2, 3 \), respectively. For the rays \( j = 4, 5 \) there are only the metatarsal-cuboid and calcaneocuboid joints, with \( n = 1, 2 \), respectively, for these two. For each ray, the moment on each joint due to \( V_j \) and \( U_j \) is

\[
M_{n,j} = \left( x_{oj} - x_j(s) \right) V_j - y_j(s) U_j.
\]

The total moment on the talonavicul joint is \( M_{13} + M_{23} + M_{33} \), resulting from the forces at the ends of the first three metatarsals, while the total moment on the calcaneocuboid joint is \( M_{14} + M_{24} + M_{34} + M_{45} + M_{55} \), due to the forces at the ends of the fourth and fifth metatarsals.

If a joint rotates through an angle \( \theta_{n,j} \), the metatarsal head will deflect through a horizontal distance \( \theta_{n,j} y_{oj} \) and through a vertical distance \( \theta_{n,j} (x_{oj} - x_j) \). The angle \( \theta_{n,j} \) is taken to be proportional to the total moment acting at that joint, and the proportionality constant for each joint is denoted by \( a_{n,j} \). Clearly \( a_{13} = a_{23} = a_{33} + a_{34} = a_{32} \). The total horizontal and vertical deflections of the metatarsal heads, \( \bar{u}_j \) and \( \bar{v}_j \), respectively, resulting from bending at all of the joints,
A biomechanical model of the foot is given in terms of the applied forces by

\[ u_j = \sum_{k=1}^{5} \left[ a_{jk} V_k + b_{jk} U_k \right] \]  

(9)

\[ v_j = \sum_{k=1}^{5} \left[ c_{jk} V_k + d_{jk} U_k \right], \]  

(10)

where \( a_{jk}, b_{jk}, c_{jk}, d_{jk} \) are the components of 5 x 5 matrices \( A, B, C, D \), respectively. Rather than write out each of these matrices, we can define them more efficiently by first introducing a single matrix in terms of elements \( \xi_{pq}, \eta_{pq} \)

\[
\begin{bmatrix}
+ \xi_{11} & \xi_{12} & \xi_{13} & 0 & 0 \\
+ \xi_{21} & \xi_{22} & \xi_{23} & 0 & 0 \\
+ \xi_{31} & \xi_{32} & \xi_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The matrices \( A, B, C, D \) can be described in the matrix form these equations can be written in the form

\[
\begin{align*}
\tilde{u}_j &= \sum_{k=1}^{5} a_{jk}^m V_k + \sum_{k=1}^{5} b_{jk}^m U_k \\
\tilde{v}_j &= \sum_{k=1}^{5} c_{jk}^m V_k + \sum_{k=1}^{5} d_{jk}^m U_k
\end{align*}
\]

(11)

(12)

where \( a_{jk}^m, b_{jk}^m, c_{jk}^m, d_{jk}^m \) are the components of 5 x 5 diagonal matrices with diagonal elements given by the integrals in equations (7) and (8). Defining the matrices \( A_{ij} = a_{ij} + a_{ij}^m, B_{ij} = b_{ij} + b_{ij}^m, C_{ij} = c_{ij} + c_{ij}^m, D_{ij} = d_{ij} + d_{ij}^m \), the total deflection of the metatarsal heads under the action of the forces \( U_k, V_k \) can be written as

\[
\begin{align*}
u_j &= \sum_{k=1}^{5} \left[ A_{jk} V_k + B_{jk} U_k \right] \\
v_j &= \sum_{k=1}^{5} \left[ C_{jk} V_k + D_{jk} U_k \right].
\end{align*}
\]

(13)

(14)

This can be expressed in terms of the forces \( R_j \) and \( T_j \) as

\[
\begin{bmatrix}
+ \xi_{11} & \xi_{12} & \xi_{13} & 0 & 0 \\
+ \xi_{21} & \xi_{22} & \xi_{23} & 0 & 0 \\
+ \xi_{31} & \xi_{32} & \xi_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The derivation of this result and the definition of the matrices \( A_{jk}, C_{jk} \) and the governing equations are given in the Appendix. Substituting equation (15) into equation (4) yields five equations for the unknown forces \( T_j, R_j \)

\[
\begin{align*}
u_j &= \sum_{k=1}^{5} \left[ A_{jk} R_k + B_{jk} T_k \right] \\
v_j &= \sum_{k=1}^{5} \left[ C_{jk} R_k + D_{jk} T_k \right].
\end{align*}
\]

(15)

(16)

The matrices \( A_{jk}, C_{jk} \) are also given in the Appendix. When equations (15) and (16) are used in equation (3), the remaining equations for the unknown forces are obtained. After some matrix manipulations, these equations can be written in the form

\[
\begin{align*}
\sum_{i=1}^{5} \left[ A_{ik}^m R_i + B_{ik}^m T_i \right] &= -v_k \theta_k \quad (k = 1-5)
\end{align*}
\]

(17)

The derivation of this result and the definition of the matrices \( A_{ik}, B_{ik} \) are also given in the Appendix. When equations (15) and (16) are used in equation (3), the remaining equations for the unknown forces are obtained. After some matrix manipulations, these equations can be written in the form

\[
\begin{align*}
\sum_{i=1}^{5} \left[ \Phi_{ik} - \Phi_{ik+1} \right] R_i + \left[ \Psi_{ik} - \Psi_{ik+1} \right] T_i \\
&= \Delta_j - \Delta_{j+1}, \quad i = 1, 2, 3, 4.
\end{align*}
\]

(18)

The derivation of this result and the definition of the matrices \( \Phi_{ik}, \Psi_{ik} \) are given in the Appendix. Defining the 11 x 1 column vectors

\[
x^T = [ R_1, R_2, R_3, R_4, T_1, T_2, T_3, T_4, T_5, M ],
\]

(19)
and
\[ s^T = [-v_1 \theta_1, -v_2 \theta_2, -v_3 \theta_3, -v_4 \theta_4, -v_5 \theta_5, \\
\Delta_1 - \Delta_2, \Delta_2 - \Delta_3, \Delta_3 - \Delta_4, \Delta_4 - \Delta_5, \\
-x_R R_0, 1 - R_0], \]
where the superscript $T$ denotes transpose, the system of eleven governing equations can be written as the single matrix equation
\[ Nx = s, \]  \hspace{1cm} (21)
where the $11 \times 11$ matrix $N$ is defined in partition form as
\[
\begin{pmatrix}
A^* & B^* & 0 & \Phi & \Psi & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]  \hspace{1cm} (22)
Here $A^*$ and $B^*$ are $5 \times 5$ matrices with components $A^*_{ij}$, $B^*_{ij}$, respectively; $\Phi$, $\Psi$, are, respectively, the $4 \times 5$ matrices with components $\Phi_{ij} - \Phi_{j+1,i}$, $\Psi_{ij} - \Psi_{j+1,i}$, $j = 1-4$, $i = 1-5$; $d = [a_1, d_2, d_3, d_4, d_5]$; $l = [1, 1, 1, 1, 1]$; and $0$ denotes various zero matrices of appropriate dimension.

**DATA**

A complete and reliable set of anatomical and biomechanical data necessary to complete equation (21) is not available at the present time. To guide future experimental studies directed toward obtaining this data and completing our biomechanical model of the foot, we will investigate the degree of sensitivity of the solutions to small changes in the data. We will also assess the relative importance of metatarsal deformation, joint deformation, and support by the plantar aponeurosis to the distribution of loading among the metatarsal heads, for various ranges of values for the data.

Coordinates for the metatarsal heads, $x_{ij}$, and for each joint, $(x_{mj}, y_{mj})$, were obtained from measurements on partially dissected amputated and cadaver feet. Lengths $I_{ij}$ for each metatarsal were also measured. For each of the metatarsals we obtained an approximation for the non-dimensional curvature, $\kappa_j$, so that the effect of curvature on the deformation of metatarsals can be assessed. For the present, variation in curvature along the length of the bone is neglected. With this data, it can be shown that
\[
x_j(s) = \frac{1}{\kappa_j} \cos (\mu_j - \kappa_j s) + x_j,
\]
\[
y_j(s) = \frac{1}{\kappa_j} \sin (\mu_j - \kappa_j s) + y_j,
\]  \hspace{1cm} (23)
The centers of these arcs are given by
\[
x_{ij} = -(B_j + \sqrt{B_j^2 - 4A_j C_j})/2A_j,
\]
\[
y_{ij} = a_j x_{ij} + b_j,
\]  \hspace{1cm} (24)
where
\[
a_j = -(x_{1j} - x_{0j})/y_{1j},
\]
\[
b_j = (x_{2j} - x_{y_j} - y_{y_j})/(2y_{4j} - 2y_{1j} - y_{0j}),
\]
\[
A_j = 1 + a_j^2, B_j = 2a_j b_j - 2x_{0j},
\]
\[
C_j = x_{0j}^2 + b_j^2 - (1/\kappa_j^2), \mu_j = \arcsin [\kappa_j (y_{1j} - y_{ij})].
\]  \hspace{1cm} (25)
For mathematical consistency, the length of the arcs from $(X_{1j}, y_{1j})$ to $(X_{0j}, 0)$, and not the measured lengths, must be used for $I_j$, so that
\[
l_j = \{\arcsin [\kappa_j (y_{1j} - y_{ij})] \}
\]
\[
- \arcsin [\kappa_j (x_{0j} - x_{ij})]/\kappa_j.
\]  \hspace{1cm} (26)

To take into account the variation in cross-sectional area and moment of inertia along the length of the metatarsals, it was assumed that the outside and inside diameter of each metatarsal could be approximated, respectively, by the formulae
\[
d_{0j} = a_{0j} + b_{0j} s + c_{0j} s^2, \]
\[
d_{ij} = a_{ij} + b_{ij} s + c_{ij} s^2, \]  \hspace{1cm} (27)
The coefficients in these equations were found by cutting each metatarsal at three locations along its length and measuring the outside and inside diameters there. The value of $s$ corresponding to each of these cuts was calculated from the measured distance $s_m$ of the cut from the metatarsal base using the formula $s/l_j = s_m/l_{ij}$. Therefore, for example, a cut made midway along the bone would correspond to $s = 0.5$. The area and moment of inertia can readily be calculated from equation (27) for each metatarsal.

To complete the data necessary to determine the deformations of the metatarsals, it is necessary to obtain a value of Young's modulus for these bones. Although no data seems to be available specifically for metatarsals, values measured for the various bones of the arm and leg show little differences (see, for example, Yamada, 1970), and so a value in this range can reasonably be chosen.

The deflections due to bending of the joints requires not only the coordinates $(x_{mj}, y_{mj})$, but also the coefficients $x_{nj}$ that relate the bending of each joint to the moment applied there. These numbers must be determined from future experimental studies. They can be found by removing all supporting structures except the ligaments from each joint and then measuring the moment necessary to bend the joint through a given deflection. Alternatively, these coefficients can be determined from the properties of the ligaments and a detailed quantitative anatomical study of their structure at each joint, similar to that done by Minami et al.
A biomechanical model of the foot (1984) for the metacarpophalangeal joint. Assume, for example, that for a given joint there are \( N \) ligaments, each with non-dimensional length \( l_i \) and cross-sectional area \( A_i \), \( i = 1-N \), and each is oriented about the joint such that it is at a non-dimensional distance \( d_i \) from the point of rotation of the joint (see Fig. 6). For that specific joint, it then follows that

\[
\begin{align*}
\mathbf{z}_{aj} &= \frac{W}{L^2 E_i} \sum_{i=1}^{N} \frac{l_i}{A_i d_i} \\
\end{align*}
\]

where \( E_i \) is Young’s modulus for ligaments.

The non-dimensional constants \( k_j \) in equation (4) can be written in terms of Young’s modulus for the plantar aponeurosis, \( E_{pa} \), according to the formula

\[
k_j = \frac{E_{pa} l_j A_{pa}}{W l_{pa}} \]

where \( A_{pa} \) and \( l_{pa} \) are the non-dimensional cross-sectional area and total length of the portion of the plantar aponeurosis connecting to the \( j \)th ray. The constants \( v_j \) in equation (4) are obtained by noting that when the metatarsals are rotated through an angle \( \theta_j \) the plantar aponeurosis is stretched an amount \( \delta_j \theta_j \), where \( \delta_j \) is the distance from the point of rotation of the metatarsophalangeal joint to where the plantar aponeurosis attaches to the phalange. Referring to Fig. 4, it can be deduced that

\[
v_j = \frac{E_{pa} \delta_j L \min}\] A_{pa} \min W l_{pa}. \]

Estimates for the lengths and cross-sectional areas for the plantar aponeurosis were made from dissected samples. Although there do not appear to be any direct experiments from which Young’s modulus for the plantar aponeurosis can be determined, a great deal of testing has been done on a variety of ligaments (for example, Woo, 1982; Woo et al., 1982; Piziali et al., 1980; Allard, 1982). We choose a value for \( E_{pa} \) that was estimated from these studies.

### Numerical Results and Discussion

For the numerical examples, we assume a total loading \( W = 350 \) N on a foot of length \( L = 24.9 \) cm. A Young’s modulus for the metatarsal bones of \( 18.5 \times 10^9 \) Pa and for the plantar aponeurosis of \( 2.5 \times 10^4 \) N cm\(^{-2} \) are used. The measured lengths, \( l_{pa} \), of the five metatarsals, \( j = 1-5 \), are, in cm, 5.90, 7.20, 7.00, 7.00, 6.90, respectively. The curvatures are all approximated by the value 0.067 cm\(^{-1} \). Approximate values for the outside and inside diameters of the metatarsals were assigned at three locations along each of these bones, in order to include the influence of variable cross-sectional area and moment of inertia. These diameters, and the locations \( s_i \) they correspond to, are given, in cm, by the three \( 3 \times 5 \) matrices \( D_{OUT}, D_{IN} \) and \( S \), respectively, where

\[
\begin{align*}
D_{OUT} &= \begin{pmatrix}
1.80 & 1.37 & 1.19 & 1.12 & 1.14 \\
1.65 & 1.07 & 1.09 & 1.02 & 1.02 \\
1.80 & 1.39 & 1.19 & 1.12 & 1.14
\end{pmatrix} \\
D_{IN} &= \begin{pmatrix}
1.60 & 1.17 & 0.99 & 0.92 & 0.94 \\
1.45 & 0.87 & 0.89 & 0.82 & 0.82 \\
1.60 & 1.17 & 0.99 & 0.92 & 0.94
\end{pmatrix} \\
S &= \begin{pmatrix}
2.95 & 3.60 & 3.50 & 3.50 & 3.45 \\
4.40 & 5.60 & 5.40 & 5.40 & 5.30
\end{pmatrix}
\end{align*}
\]

The five columns correspond to the metatarsals one to five, respectively. The measured transverse locations of the five metatarsal heads, \( z_j \) (Fig. 3), are, in cm, 0, 2.60, 4.32, 5.76, 6.92, and the distances \( d_{jj} \), that determine the amount of stretch the plantar aponeurosis undergoes under the windlass effect (equation 30 and Fig. 4) are, for rays 1-5, respectively, 2.0, 1.7, 1.5, 1.3, 1.3. The cross-sectional area for the plantar aponeurosis along each ray is approximated by the value 0.1 cm\(^2\). Measurements of the \( y_j \)-coordinate of the point of application of loading on the talus, \( y_{wj} \), varied negligibly among the rays, and a value of 4.02 cm is used. The moment arm for \( R_{w} \), \( x_{w} \), was measured to be \(-3.18\) cm. The \( x_j \) and \( y_j \) location of each of the joints are given in cm by the non-rectangular arrays \( A_x \) and \( A_y \), respectively, in which the columns represent the rays one to five, respectively, and the rows represent the joints.

\[
\begin{align*}
A_x &= \begin{pmatrix}
13.71 & 14.61 & 13.95 & 12.66 & 11.80 \\
7.70 & 7.14 & 6.99 & 5.77 & 5.07 \\
4.49 & 5.26 & 4.87 & 2.48 & 2.48 \\
3.44 & 3.44 & 3.44
\end{pmatrix} \\
A_y &= \begin{pmatrix}
3.40 & 3.31 & 3.06 & 2.34 & 2.06 \\
3.63 & 3.73 & 3.59 & 3.32 & 3.32 \\
4.55 & 4.55 & 4.55
\end{pmatrix}
\end{align*}
\]

We will present solutions for the case where the heel is down (\( h = 0 \)) and supporting no load (\( R_h = 0 \)), and the case where the heel is elevated a distance of 10 cm. The coefficients for joint deflection, \( z_{nj} \), are all assumed equal; i.e. \( a_{nj} = \alpha \) for all \((n, j)\). The relative importance of joint deflection and metatarsal deformation to the displacement of the metatarsal heads under applied loads can be compared by examining the deformation matrices \( A, B, C, D \) of equations (13), (14) under the assumption first that the metatarsals are flexible and the joints are rigid and second that the metatarsals are rigid and the joints are flexible. Since these matrices

---

Fig. 6. Bending of a joint is resisted by attached ligaments.
determine the deflection of the metatarsal heads for given applied loads, if their components are the same order of magnitude in both cases, it follows that the effect of metatarsal deformation and of joint deflection are of equal importance. In the first case $\alpha = 0$, and these matrices reduce to diagonal form with numerical values given by

$$A = \text{Diag}[-0.0023, -0.0062, -0.0065, -0.0036, -0.0020],$$

$$B = \text{Diag}[-0.0013, -0.0049, -0.0050, -0.0037, -0.0028],$$

$$C = \text{Diag}[0.0070, 0.0151, 0.0196, 0.0125, 0.0076],$$

$$D = \text{Diag}[0.0023, 0.0062, 0.0065, 0.0036, 0.0020].$$

In the second case matrices with values of the same order of magnitude as these are obtained when $\alpha = \pi/180$, corresponding to one degree of joint bending under an applied moment equal to $W \times L$. Assuming rigid metatarsals gives

$$A = \begin{bmatrix}
0.0028 & 0.0014 & 0.0013 & 0 & 0 \\
0.0013 & 0.0031 & 0.0013 & 0 & 0 \\
0.0013 & 0.0014 & 0.0029 & 0 & 0 \\
0 & 0 & 0 & 0.0014 & 0.0009 \\
0 & 0 & 0 & 0.0010 & 0.0013
\end{bmatrix},$$

$$B = \begin{bmatrix}
-0.0013 & -0.0006 & -0.0006 & 0 & 0 \\
-0.0006 & -0.0013 & -0.0006 & 0 & 0 \\
-0.0006 & -0.0006 & -0.0012 & 0 & 0 \\
0 & 0 & 0 & -0.0005 & -0.0003 \\
0 & 0 & 0 & -0.0003 & -0.0004
\end{bmatrix},$$

$$C = \begin{bmatrix}
0.0064 & 0.0032 & 0.0030 & 0 & 0 \\
0.0032 & 0.0075 & 0.0033 & 0 & 0 \\
0.0030 & 0.0033 & 0.0068 & 0 & 0 \\
0 & 0 & 0 & 0.0043 & 0.0027 \\
0 & 0 & 0 & 0.0027 & 0.0037
\end{bmatrix},$$

$$D = \begin{bmatrix}
-0.0028 & -0.0013 & -0.0013 & 0 & 0 \\
-0.0014 & -0.0031 & -0.0014 & 0 & 0 \\
-0.0013 & -0.0013 & -0.0029 & 0 & 0 \\
0 & 0 & 0 & -0.0014 & 0.0010 \\
0 & 0 & 0 & 0.0009 & -0.0013
\end{bmatrix}.$$

Negative values for the forces $T_j$ are not physically meaningful, and when they occur a new solution must be obtained assuming these values are zero. Since the $T_j$ are virtually zero in the above cases, such a procedure is not required here, as it would have negligible effect on the values obtained for $R_j$ and $M$. When the heel is up the windlass mechanism induces significant force on the plantar aponeurosis. For a heel elevation of 10 cm, the altered load distribution is shown in Fig. 7b as curves 4, 5 and 6, corresponding to curves 1, 2, 3 respectively, in Fig. 7a, and the forces $T_j$ and moment $M$ are listed in Table 1 as cases 4, 5, 6, respectively.

For the three cases illustrated in Fig. 7a, deflection of the metatarsal heads is due almost entirely to $R_j$, since $T_j \approx 0$. Also, since $\theta_j = 0$, it follows from equation (5) that $V_j = R_j$ and $U_j = T_j$. In addition, the vertical displacements are equal to $v_j$ when $\theta_j = 0$. Therefore the distribution of the loading among the five metatarsals in these cases is determined entirely by the values of the deformation matrix $C$. Curve 1 in Fig. 7a can be correlated with the values of $C$ corresponding to the first case, flexible metatarsals and rigid joints. The smaller the value of an element in this matrix, the greater the resistance of the corresponding metatarsal to vertical displacement under vertical load, and therefore the greater the share of the total load borne by that metatarsal. The values are affected by the cross-sectional area and moment of inertia of the corresponding metatarsal, and by the metatarsal's orientation, which influences the relative importance of bending and axial compression.

Each of the diagonal terms of the matrix $C$ for the
A biomechanical model of the foot

Fig. 7. Fraction of the total load borne by each metatarsal. Left: heel down position. 1: flexible metatarsals, rigid joints; 2: rigid metatarsals, flexible joints; 3: flexible metatarsals, flexible joints. Right: heel up position. 4: flexible metatarsals, rigid joints; 5: rigid metatarsals, flexible joints; 6: flexible metatarsals, flexible joints.

Table 1. The force applied by the plantar aponeurosis to each metatarsal, and the moment $M$, for various cases described in the text

<table>
<thead>
<tr>
<th>Case</th>
<th>Metatarsal 1</th>
<th>Metatarsal 2</th>
<th>Metatarsal 3</th>
<th>Metatarsal 4</th>
<th>Metatarsal 5</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.007</td>
<td>-0.007</td>
<td>-0.006</td>
<td>0.528</td>
</tr>
<tr>
<td>2</td>
<td>0.009</td>
<td>0.008</td>
<td>0.008</td>
<td>0.007</td>
<td>0.008</td>
<td>0.521</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.002</td>
<td>-0.001</td>
<td>0.524</td>
</tr>
<tr>
<td>4</td>
<td>0.522</td>
<td>0.387</td>
<td>0.369</td>
<td>0.379</td>
<td>0.432</td>
<td>0.297</td>
</tr>
<tr>
<td>5</td>
<td>0.535</td>
<td>0.407</td>
<td>0.389</td>
<td>0.400</td>
<td>0.453</td>
<td>0.334</td>
</tr>
<tr>
<td>6</td>
<td>0.521</td>
<td>0.381</td>
<td>0.364</td>
<td>0.381</td>
<td>0.434</td>
<td>0.311</td>
</tr>
<tr>
<td>7</td>
<td>0.076</td>
<td>0.067</td>
<td>0.072</td>
<td>0.060</td>
<td>0.067</td>
<td>0.523</td>
</tr>
<tr>
<td>8</td>
<td>0.062</td>
<td>0.062</td>
<td>0.072</td>
<td>0.063</td>
<td>0.074</td>
<td>0.519</td>
</tr>
<tr>
<td>9</td>
<td>0.047</td>
<td>0.056</td>
<td>0.071</td>
<td>0.067</td>
<td>0.080</td>
<td>0.515</td>
</tr>
<tr>
<td>10</td>
<td>0.076</td>
<td>0.058</td>
<td>0.059</td>
<td>0.050</td>
<td>0.058</td>
<td>0.523</td>
</tr>
<tr>
<td>11</td>
<td>0.065</td>
<td>0.054</td>
<td>0.057</td>
<td>0.053</td>
<td>0.063</td>
<td>0.520</td>
</tr>
<tr>
<td>12</td>
<td>0.054</td>
<td>0.050</td>
<td>0.054</td>
<td>0.057</td>
<td>0.067</td>
<td>0.516</td>
</tr>
<tr>
<td>13</td>
<td>0.091</td>
<td>0.072</td>
<td>0.073</td>
<td>0.057</td>
<td>0.060</td>
<td>0.527</td>
</tr>
<tr>
<td>14</td>
<td>0.105</td>
<td>0.078</td>
<td>0.074</td>
<td>0.054</td>
<td>0.053</td>
<td>0.531</td>
</tr>
<tr>
<td>15</td>
<td>0.087</td>
<td>0.063</td>
<td>0.062</td>
<td>0.046</td>
<td>0.053</td>
<td>0.526</td>
</tr>
<tr>
<td>16</td>
<td>0.098</td>
<td>0.067</td>
<td>0.065</td>
<td>0.043</td>
<td>0.049</td>
<td>0.529</td>
</tr>
<tr>
<td>17</td>
<td>0.530</td>
<td>0.403</td>
<td>0.385</td>
<td>0.397</td>
<td>0.449</td>
<td>0.333</td>
</tr>
<tr>
<td>18</td>
<td>0.511</td>
<td>0.396</td>
<td>0.384</td>
<td>0.400</td>
<td>0.457</td>
<td>0.324</td>
</tr>
<tr>
<td>19</td>
<td>0.492</td>
<td>0.389</td>
<td>0.382</td>
<td>0.404</td>
<td>0.466</td>
<td>0.315</td>
</tr>
<tr>
<td>20</td>
<td>0.520</td>
<td>0.373</td>
<td>0.355</td>
<td>0.380</td>
<td>0.437</td>
<td>0.326</td>
</tr>
<tr>
<td>21</td>
<td>0.506</td>
<td>0.368</td>
<td>0.351</td>
<td>0.384</td>
<td>0.443</td>
<td>0.318</td>
</tr>
<tr>
<td>22</td>
<td>0.492</td>
<td>0.362</td>
<td>0.347</td>
<td>0.388</td>
<td>0.448</td>
<td>0.311</td>
</tr>
<tr>
<td>23</td>
<td>0.549</td>
<td>0.411</td>
<td>0.387</td>
<td>0.393</td>
<td>0.440</td>
<td>0.342</td>
</tr>
<tr>
<td>24</td>
<td>0.568</td>
<td>0.418</td>
<td>0.389</td>
<td>0.390</td>
<td>0.432</td>
<td>0.351</td>
</tr>
<tr>
<td>25</td>
<td>0.534</td>
<td>0.379</td>
<td>0.358</td>
<td>0.376</td>
<td>0.431</td>
<td>0.333</td>
</tr>
<tr>
<td>26</td>
<td>0.549</td>
<td>0.385</td>
<td>0.362</td>
<td>0.371</td>
<td>0.426</td>
<td>0.340</td>
</tr>
<tr>
<td>27</td>
<td>0.338</td>
<td>0.302</td>
<td>0.327</td>
<td>0.273</td>
<td>0.303</td>
<td>0.533</td>
</tr>
<tr>
<td>28</td>
<td>0.505</td>
<td>0.390</td>
<td>0.371</td>
<td>0.385</td>
<td>0.434</td>
<td>0.332</td>
</tr>
</tbody>
</table>

second case, flexible joints and rigid metatarsals, is a measure of the vertical displacement experienced by the corresponding metatarsal head as a result of a vertical force acting upon it. The difference in values among $C_{11}$, $C_{22}$, $C_{33}$, and between $C_{44}$ and $C_{55}$, is due almost entirely to the different lengths of the metatarsals. The values for $C_{44}$ and $C_{55}$ are smaller than those for $C_{11}$, $C_{22}$ and $C_{33}$, since the corresponding
rays have only two joints, and are therefore more rigid than the other three. The off-diagonal terms in $C$ measure the vertical displacement of a given metatarsal head due to a vertical force acting on a different metatarsal head. Since these terms are nearly equal within the two sub-arrays, it is possible to correlate the results shown by curve 2 in Fig. 7a with the values along the diagonal of the matrix $C$.

The third case is a combination of the first two cases, and the load supported by each metatarsal as illustrated by curve 3 in Fig. 7a has a value between the values corresponding to curves 1 and 2. When the heel is raised, all four deformation matrices must be considered in determining the distribution of loading among the metatarsal heads. A simple correlation between the values of these matrices and the results shown in Fig. 7b, for example, is therefore not possible.

When the metatarsals are rigid and the effect of the plantar aponeurosis is neglected ($E_p = 0$), the deflections of the metatarsal heads are directly proportional to $\alpha$, the load distribution is independent of $\alpha$, and the solution is given approximately by curve 2, since that solution is only slightly influenced by the plantar aponeurosis. However, the large deflections that occur when the joints are very flexible ($\alpha$ large) are resisted by the plantar aponeurosis and substantial alteration occurs in the load distribution from that given by curve 2. For example, when $\alpha = 100\pi/180$, deflection of the metatarsal heads of up to 1 cm occurs, resulting in large forces applied to the metatarsal heads by the plantar aponeurosis. Assuming rigid metatarsals, when $h = 0$ the forces $T_j$ applied by the plantar aponeurosis to the metatarsals are listed in Table 1, along with the moment $M$, as case 27, and when the heel is elevated, the corresponding values are listed as case 28.

All the above examples correspond to $\beta = 0$. The effect of pronation and supination can be examined by obtaining the solutions for various positive and negative values of $\beta$. In the following examples, we assume that $\alpha = 10\pi/180$. When the heel is down ($h = 0$), $\beta = 0$ and the metatarsals are rigid, the support distribution is shown as curve 7 in Fig. 8a. One degree of supination ($\beta = \pi/180 = 0.01745$) changes the distribution to curve 8, and increasing the supination to $2^\circ$ ($\beta = 0.0349$) changes it to curve 9. In the latter case, the extreme shifting of the load onto the fifth metatarsal decreases the load borne by the fourth metatarsal below that of the third, which assumes the largest portion of the total load borne by the first three metatarsals. Corresponding results for these three cases when deformation of the metatarsals is included are shown as curves 10, 11, and 12, respectively, in Fig. 8b. One and two degrees of pronation ($\beta = -0.01745$ and $\beta = -0.0349$, respectively) are shown in Fig. 9a as curves 13 and 14, respectively. The $\beta = 0$ case is repeated as curve 7 for comparison. For large pronation (curve 14) the loading on the first three metatarsals is shifted toward the first, and the loading on the fourth and fifth metatarsals, while still large, is shifted toward the fourth. Corresponding results for

![Fig. 8. Fraction of the total load borne by each metatarsal for various degrees of supination, when the heel is down. Left: rigid metatarsals. 7: $\beta = 0$, 8: $\beta = 1^\circ$, 9: $\beta = 2^\circ$. Right: flexible metatarsals. 10: $\beta = 0^\circ$, 11: $\beta = 1^\circ$, 12: $\beta = 2^\circ$.](image)
A biomechanical model of the foot

Fig. 9. Fraction of the total load borne by each metatarsal for various degrees of pronation, when the heel is down. Left: rigid metatarsals. 7: $\beta = 0$, 13: $\beta = -1^\circ$, 14: $\beta = -2^\circ$. Right: flexible metatarsals. 10: $\beta = 0$, 15: $\beta = -1^\circ$, 16: $\beta = -2^\circ$.

These three cases when deformation of the metatarsals is included are shown in Fig. 9b. Curves 15 and 16 correspond to $1^\circ$ and $2^\circ$ of pronation, respectively, and curve 10 for $\beta = 0$ is repeated. The forces $T_i$ applied by the plantar aponeurosis, and the moment $M$, are given in Table I, with each case identified by the same number used for its corresponding graph.

Finally, we examine the effect of pronation and supination when the heel is elevated a distance of 10 cm, again for the case $\alpha = 10\pi/180$. For rigid metatarsals, and $\beta = 0$, the solution is shown in Fig. 10a as curve 17, while curves 18 and 19 correspond to $1^\circ$ and $2^\circ$ of supination, respectively. Again, the shifting of the load borne by the first three metatarsals toward the third and the load borne by the fourth and fifth metatarsals toward the fifth results in the fourth metatarsal carrying less load than the third, as shown by curves 18 and 19. Figure 10b illustrates the changes that occur when deformation of the metatarsals is included, with curve 20 corresponding to $\beta = 0$, and curves 21 and 22 corresponding to $1^\circ$ and $2^\circ$ of supination, respectively. Pronation of $1^\circ$ gives the distribution of load shown in Fig. 11a as curve 23, while increased pronation ($2^\circ$) is illustrated by...
curve 24. The case $\beta = 0$ is repeated for comparison as curve 17. Figure 11b shows corresponding results when deformation of the metatarsals is included. Curves 25 and 26 correspond to 1° and 2° of pronation, respectively, and curve 20 for $\beta = 0$ is repeated for comparison. Again, the forces $T_j$ and moment $M$ are listed in Table 1 for each case.

In each of the above cases the shifting of support due to pronation or supination is evident. The small negative value for $R_5$ in case 24 indicates that this solution is not valid, since it requires a downward force on the fifth metatarsal. Therefore a new solution should be obtained satisfying the constraint $R_5 = 0$, in which the loadings $R_1-R_4$ would be slightly altered.

The extent to which a given amount of pronation or supination shifts the load distribution among the metatarsal heads is related to the flexibility of the joints. For larger values of $\alpha$, a larger value for $\beta$ is needed to achieve the same effect. Increasing $\alpha$ also decreases the role of metatarsal deformation in determining the load distribution. The data used to determine metatarsal deformation should yield results of the correct order of magnitude. The order of magnitude of $\alpha$, and its variation among the different joints, must be determined experimentally. The solutions shown here illustrate the effect of a wide range in values for these constants. Our solutions are strongly influenced by the assumption that all the $\sigma_{nj}$ are equal. There is undoubtedly considerable variation in the stiffness of the different joints, and this would significantly affect the distribution of load among the metatarsals.

The method presented here, in which the foot is analyzed as a statically indeterminate structure, attempts to determine the support distribution of the foot in terms of its structural anatomy and biomechanical properties. The benefit of such an approach is that it can potentially relate stress distribution to the physiological state of the foot and predict the effect of clinical interventions intended to correct various conditions. The influence of repetitive loading can induce fatigue fractures in bone (Lease and Evans, 1959; Morris and Blickenstaff, 1967) and may result in such injuries as the Jones fracture, which occurs in the fifth metatarsal (Arangio, 1983). The approach to the biomechanics of the foot that we have presented can

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Fig. 11. Fraction of the total load borne by each metatarsal for various degrees of pronation, when the heel is elevated. Left: rigid metatarsals. 17: $\beta = 0$, 22: $\beta = -1^\circ$, 24: $\beta = -2^\circ$. Right: flexible metatarsals. 20: $\beta = 0$, 25: $\beta = -1^\circ$, 26: $\beta = -2^\circ$.
lead to a better understanding of the cause of a wide variety of such pathological conditions, and provide useful information for their prevention and treatment.

REFERENCES


APPENDIX

To obtain equations (15) and (16) from equations (13), (14) and (5), we let \( G_{ij}(x) \) denote the components of a 5 x 5 diagonal matrix with diagonal elements \( \lambda_1, \ldots, \lambda_5 \), so that \( G_{ij}(x) = \lambda_i \delta_{ij} \), where \( \delta_{ij} = 1 \) if \( i = j \) and \( \delta_{ij} = 0 \) if \( i \neq j \). Substituting equation (5) into equation (13), the deflections \( \epsilon \) can be written in terms of the forces \( R_i, T_i \) as

\[
\epsilon_i = \sum_{j=1}^{5} \left[ A_{ij} \epsilon_j + B_{ij} T_j \right] (A1)
\]

where

\[
A_{ij} = \sum_{k=1}^{5} \left[ C_{ij} G_{ii}(x) + D_{ij} G_{ii}(x) \right],
\]

\[
B_{ij} = \sum_{k=1}^{5} \left[ C_{ij} G_{ii}(x) + D_{ij} G_{ii}(x) \right].
\]

This completes the derivation of equations (15) and (16). Substituting equation (15) into equation (4) gives

\[
\sum_{i=1}^{5} \left[ A_{ii} R_i + B_{ii} T_i \right] = -u_j \theta_j (k = 1-5) (A3)
\]

To derive equation (18), we use equations (15), (16) in equation (3) to show that

\[
\psi_j \cos \theta_j - u_j \sin \theta_j - \Delta_j = \sum_{i=1}^{5} \left[ A_{ij} \cos \theta_j \right] C_{ij} R_i + D_{ij} T_j - \sum_{i=1}^{5} \left[ A_{ij} \sin \theta_j \right] \left( A_{ij} R_i + B_{ij} T_i \right) - \Delta_j (A5)
\]

where

\[
\phi_k = \sum_{j=1}^{5} \left[ A_{ij} \cos \theta_j \right] C_{ij} - \sum_{j=1}^{5} \left[ A_{ij} \sin \theta_j \right] D_{ij} (A6)
\]

and

\[
\psi_k = \sum_{j=1}^{5} \left[ A_{ij} \cos \theta_j \right] D_{ij} - \sum_{j=1}^{5} \left[ A_{ij} \sin \theta_j \right] B_{ij} (A7)
\]