

AVERAGE JANUARY TEMPERATURE (F°)

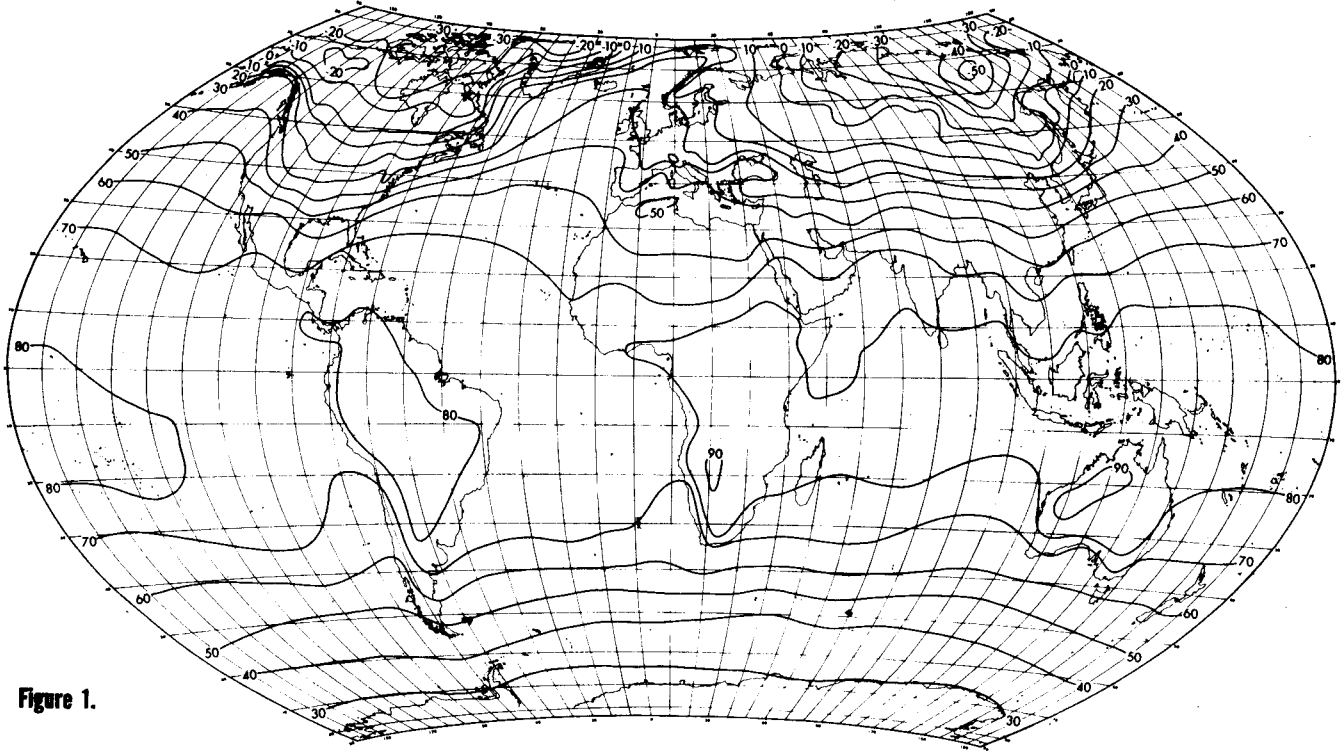


Figure 1.

AVERAGE JULY TEMPERATURE (F°)

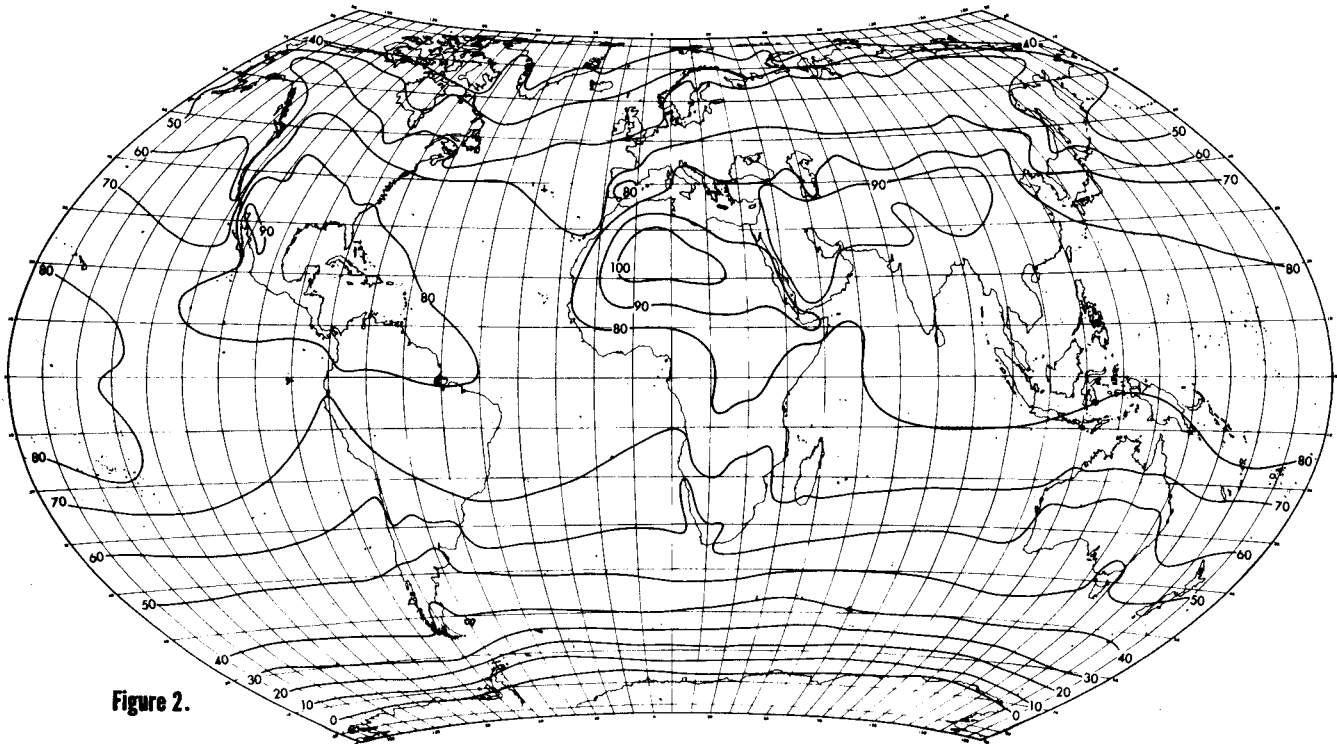
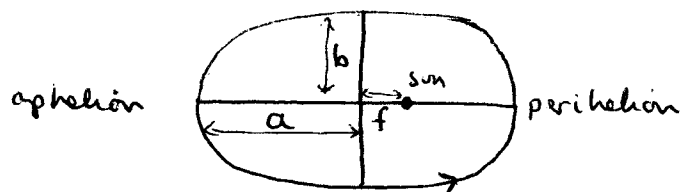


Figure 2.

DISTRIBUTION of SOLAR RADIATION as determined by
GEOMETRY OF EARTH'S ORBIT.

I planetary orbits are ellipses



a = semimajor axis

b = semi-minor axis

f = distance from center to focus. Sun is at one focus.

$$a^2 = f^2 + b^2$$

Define eccentricity $e \equiv f/a$.

($e = 0$ for a circle)

closest approach of planet to sun (perihelion) $= a - f = a(1 - e) = r_p$

greatest distance of planet from sun (aphelion) $= a + f = a(1 + e) = r_a$

	<u>Earth</u>	<u>Mars</u>
now	$e = 0.0175$	$e = 0.093$

extreme values $\left\{ \begin{array}{l} e = 0 \\ e = 0.073. \end{array} \right.$

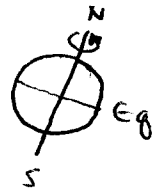
Eccentricity varies over 10^5 -year time scales due to gravitational pull of other planets.

(mainly Jupiter and Earth's moon)

II Rotation of planet.

The axis of rotation joins the North and South poles.

The circumference of the planet halfway between N, S poles is the equator.



The rotation rate is Ω .

$$\Omega = 2\pi \frac{\text{radians}}{\text{day}} = 360 \frac{\text{degrees}}{\text{day}} = 15 \frac{\text{degrees}}{\text{hour}} \text{ for earth.}$$

period of rotation is 24 hours for earth

24.5 hours Mars

10 hours Jupiter

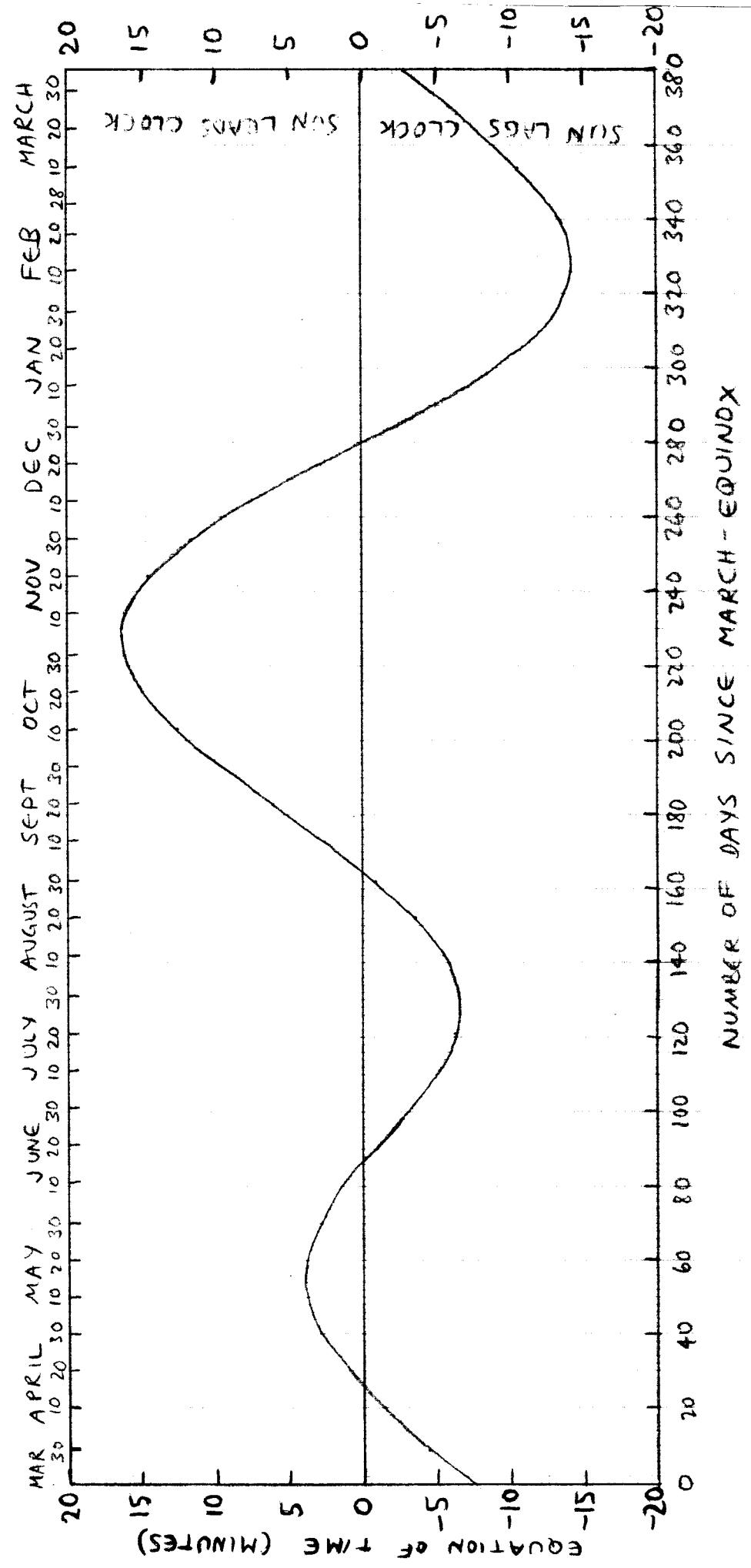
Rotation rate controls day-night cycle of insolation
and controls circulation of atmosphere.

The length of a solar day is the time from local solar noon on one day to local solar noon on the next day.

"Local solar noon" is the time that the earth-sun line intersects our longitude circle. Local sun time is the time of a clock that reads 12:00 at local solar noon.

The length of a solar day, averaged over a year, is 24 hours. Particular solar days deviate from this by up to 30 seconds, depending on time of year, because the earth's axis is tilted and because the orbit is elliptical.

EQUATION OF TIME (SUN TIME - MEAN SUN TIME)
 from STRAHLER 1975: PHYSICAL GEOGRAPHY page 83.



Cumulative deviations of solar day from 24 hours cause local sun time to deviate from mean solar time; this difference is called the "equation of time". Clock time differs from mean solar time because of the use of standard time ZONES.

There are 24 time zones, each covering 15° of longitude
 $(24 \times 15^\circ = 360^\circ)$

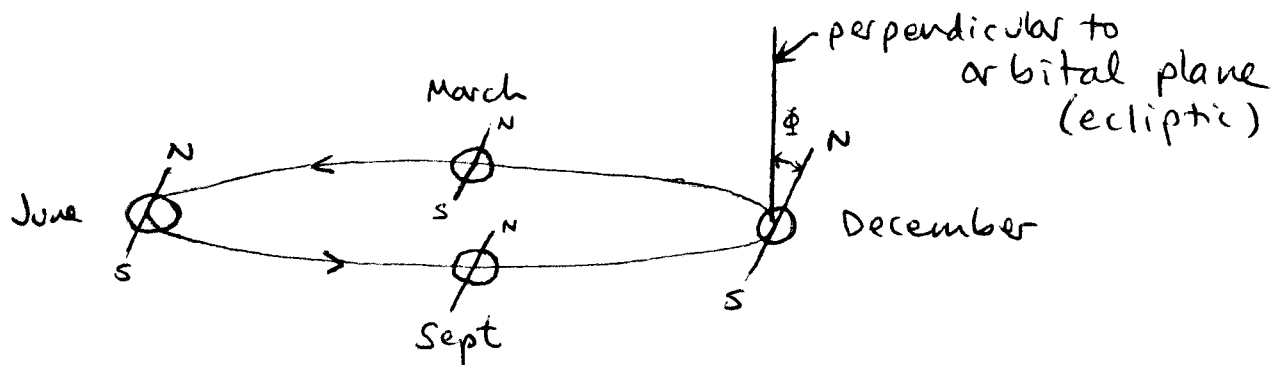
eg.	<u>time zone</u>	<u>standard (central) longitude</u>
	GMT	0°
	GMT+1	15°E
	GMT-8	120°W

Longitude of Seattle is 122°W so Seattle is in the " 120°W " time zone or the "GMT-8" time zone.

Noon GMT is 4 AM in Seattle.

III' Relation of rotation axis to the orbital plane.

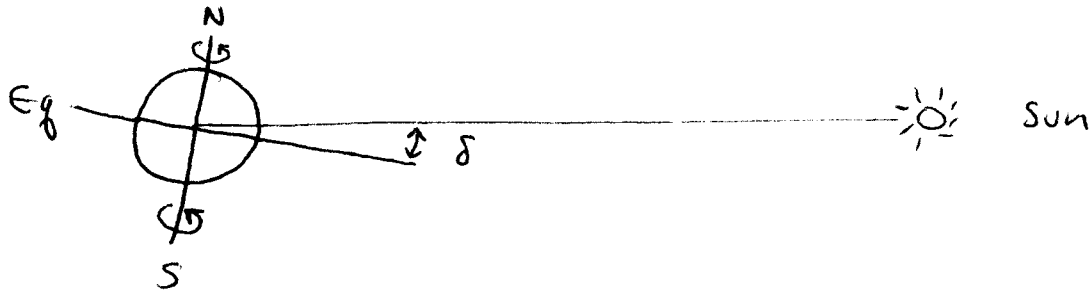
"Tilt" angle Φ is the angle between the axis of rotation and the perpendicular to the orbital plane. This tilt is responsible for the seasons.



For Earth, Φ varies in the range $22^\circ - 24.5^\circ$, on a 40,000-year cycle. It is now 23.5° and decreasing.

For Mars, Φ is now 23.9° . It varies between 15° and 35° .

Because the tilt remains constant over the year, the angle between the earth-sun line and the earth's equator varies between $+23.5^\circ$ and -23.5° . This angle is called the declination. δ

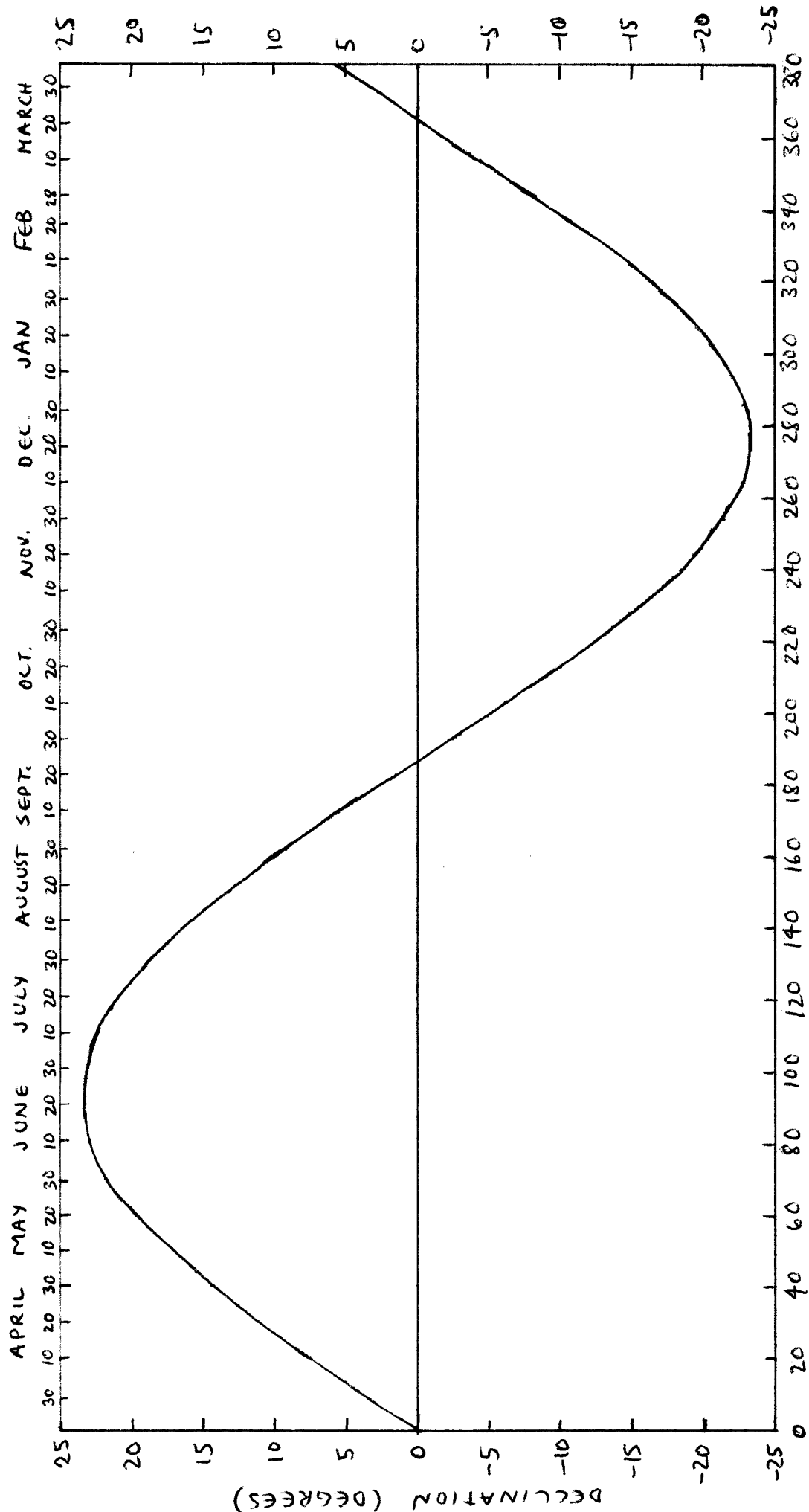


Declination thus also is the latitude where solar beam is vertical at noon.

The time of year when $\delta = 0^\circ$ is called "equinox".

The time of year when $\delta = \pm 23.5^\circ$ is called "solstice".

<u>time of year</u>	<u>declination</u>	<u>sun is vertical at what latitude ϕ</u>
June solstice (21 June)	$+23.45^\circ$	$23.45^\circ N$ (Tropic of Cancer)
September equinox (23 Sept)	0°	$0^\circ N$ (Equator)
December solstice (22 Dec)	-23.45°	$23.45^\circ S$ (Tropic of Capricorn)
March equinox (21 March)	0°	$0^\circ N$ (Equator)

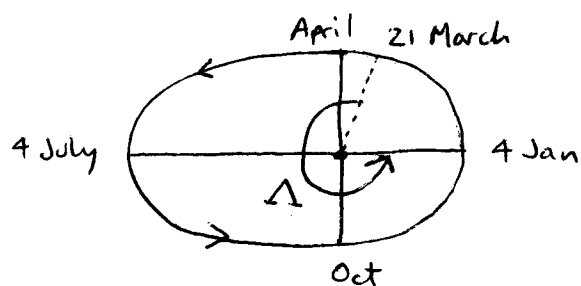


NUMBER OF DAYS SINCE MARCH-EQUINOX

IV Phase of season relative to position on elliptical orbit

"relative longitude of perihelion" Λ

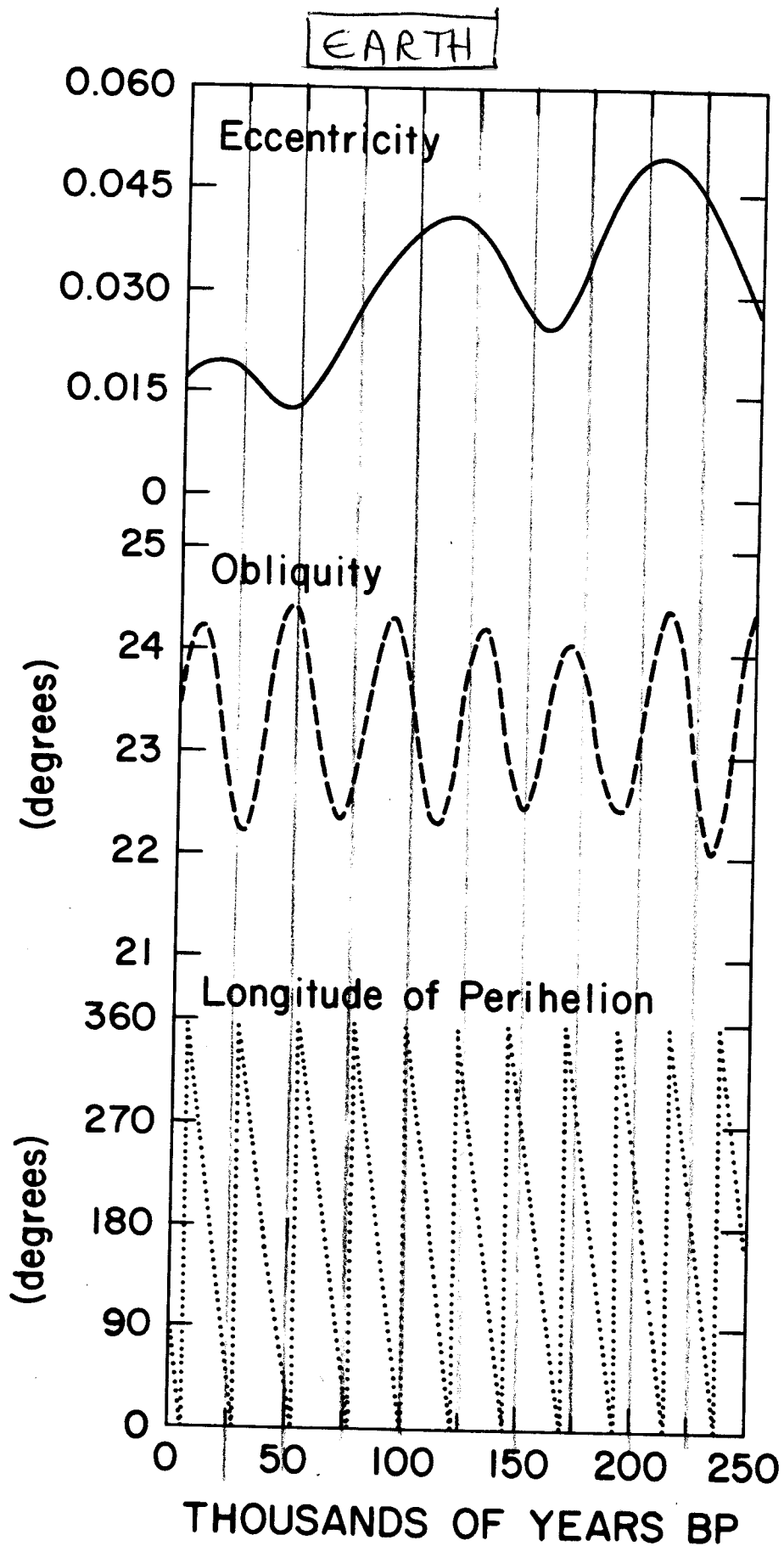
Draw the earth's orbit, looking down on the north pole of the orbit:



Λ is the angle in degrees from March-equinox (Hartmann's convention) to the time of perihelion.

Now $\Lambda \approx 282^\circ$

Time of perihelion varies throughout the year, with a period about 20,000 years.



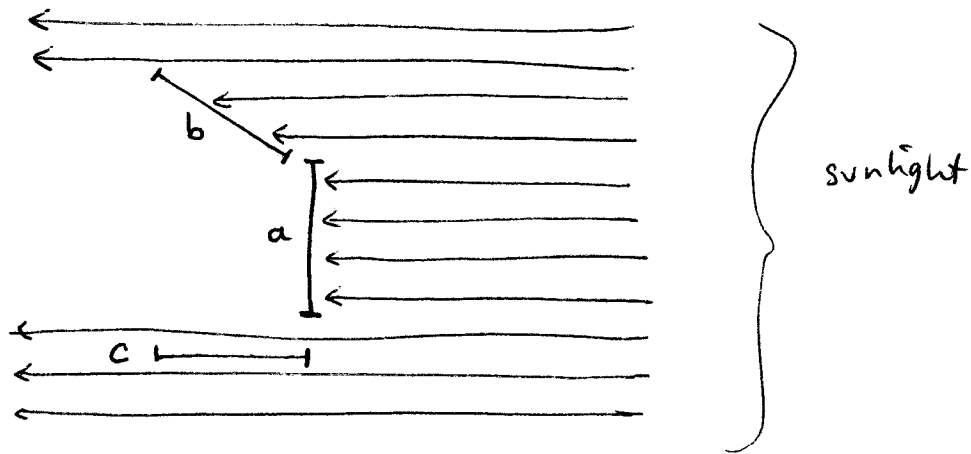
(angle
from
September-
equinox)

B. Radiation received on a surface.

The rate of energy received by a flat surface* of 1 m^2 oriented perpendicular to the solar beam is equal to the solar constant, $1370 \text{ W m}^{-2} = S_0$

* at the top of earth's atmosphere, at the mean earth-sun distance.

A tilted surface receives less than this:

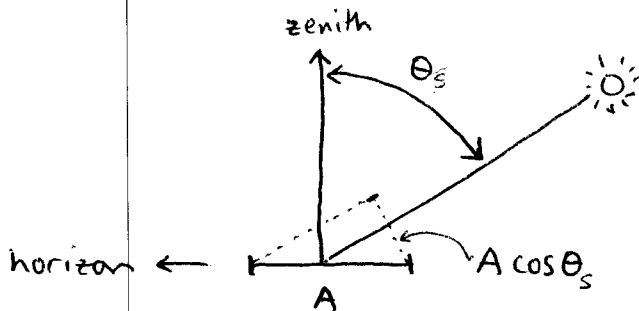


Surface a is perpendicular to the sun and intercepts S_0 .

Surface b is tilted 60° and intercepts $\frac{1}{2} S_0$.

Surface c is tilted 90° and intercepts zero.

Define the solar zenith angle θ_s as the angle from local vertical (or "zenith") down to the solar beam.



Project the surface onto a plane perpendicular to the solar beam. The area $A \text{ (m}^2\text{)}$ projects to a smaller area $A \cos \theta_s \text{ (m}^2\text{)}$.

All the sunlight that passes through this projected area is also incident on area A .

The power incident on the projected area is

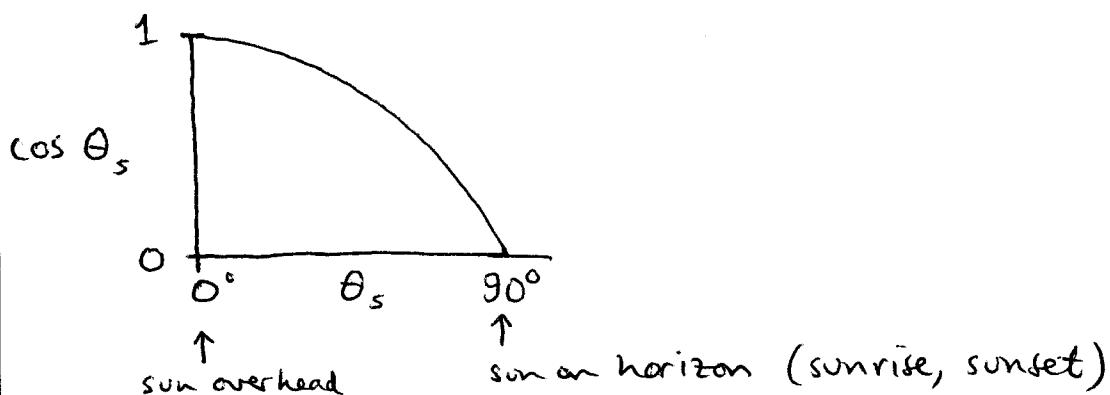
(solar constant) $\times$ (area)

$$(S_0 \text{ W m}^{-2}) \times (A \cos \theta_s \text{ m}^2) = S_0 A \cos \theta_s \text{ watts}.$$

This energy is distributed over area A , which therefore receives insolation (also called irradiance, radiation flux)

$$F = \frac{S_0 A \cos \theta_s \text{ watts}}{A \text{ m}^2} = \underline{S_0 \cos \theta_s \text{ W m}^{-2}}$$

So the flux received on a horizontal surface at the top of the atmosphere is $S_0 \cos \theta_s \text{ W m}^{-2}$



Most earth-surfaces are not $\perp$ the solar beam.

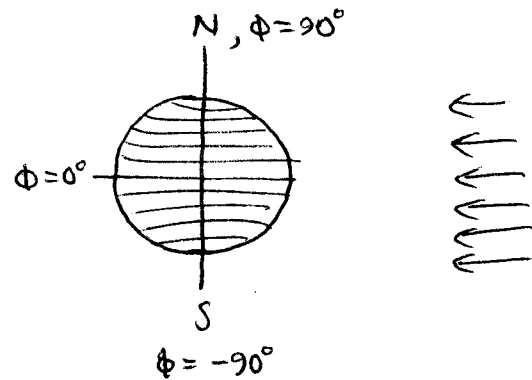
The solar beam is vertical only at one point on earth at any particular time. This point is always somewhere between latitudes 23.5°N and 23.5°S i.e. in the "tropics".

① eg at equinox, at noon.

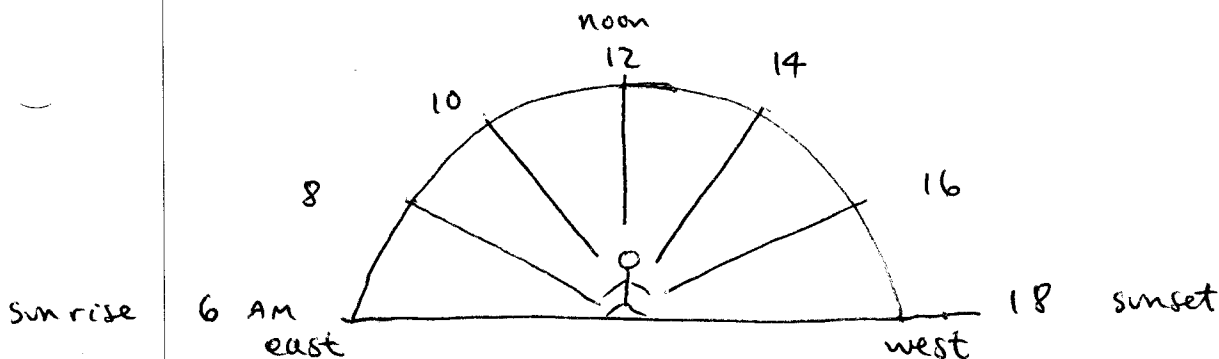
$$\theta_s = 90^\circ \text{ at poles}$$

$$\theta_s = \phi \text{ elsewhere}$$

where $\phi = \text{latitude}$

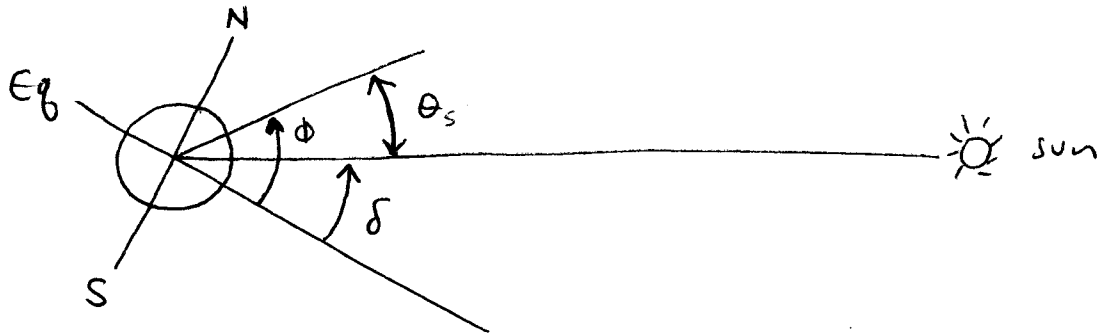


② eg at equator at equinox. (21 March, 23 Sept.)



<u>local sun time.</u>	<u>θ_s</u>	<u>$\cos \theta_s$</u>	<u>irradiance at Top of Atmosphere (T.O.A.)</u>
6	90	0	0
8	60	0.5	$S_0 / 2$
9	45	0.71	$0.71 S_0$
12	0	1	S_0
18	90	0	0

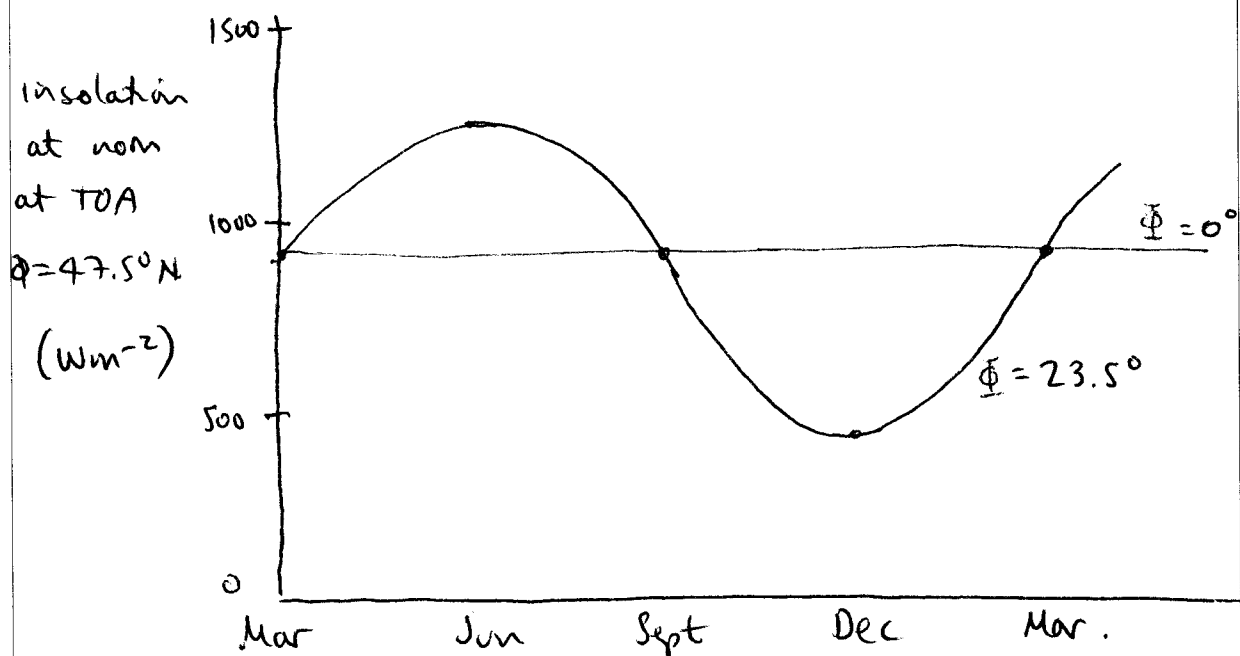
③ eg. noon, not equinox, not equator. At latitude ϕ



$\theta_s = |\phi - \delta|$ gives zenith angle at noon.

Seattle, $\phi = 47.5^\circ$.

time	δ	θ_s	insolation at TOA
noon 21 June	23.5	$47.5 - 23.5 = 24^\circ$	$1370 \cos 24 = 1252 \text{ Wm}^{-2}$
noon 22 Dec	-23.5	$47.5 + 23.5 = 71^\circ$	$1370 \cos 71 = 446 \text{ "}$
noon 21 Mar, 23 Sep	0	47.5°	$1370 \cos 47.5 = 926 \text{ "}$



A. Solar zenith angle for any latitude, date, and hour

Define: $\theta_s, \phi, \delta, h$

① θ_s = zenith angle (0° to 90°). Angle between vertical and sun.

② ϕ = latitude ($+90^\circ$ to -90°) negative in S. Hemisphere.

North Pole $\phi = +90^\circ$

Equator $\phi = 0$

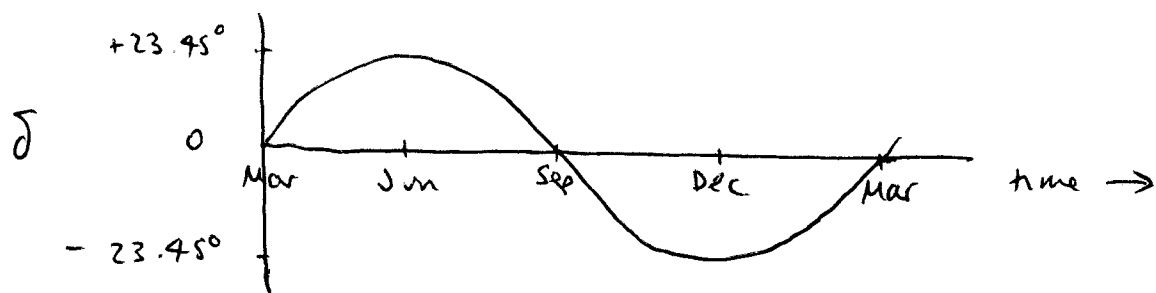
South Pole $\phi = -90^\circ$

③ δ = declination = angle between $\left\{ \begin{array}{l} \text{plane of equator} \\ \text{earth-sun line} \end{array} \right.$
($+23.45^\circ$ to -23.45°)

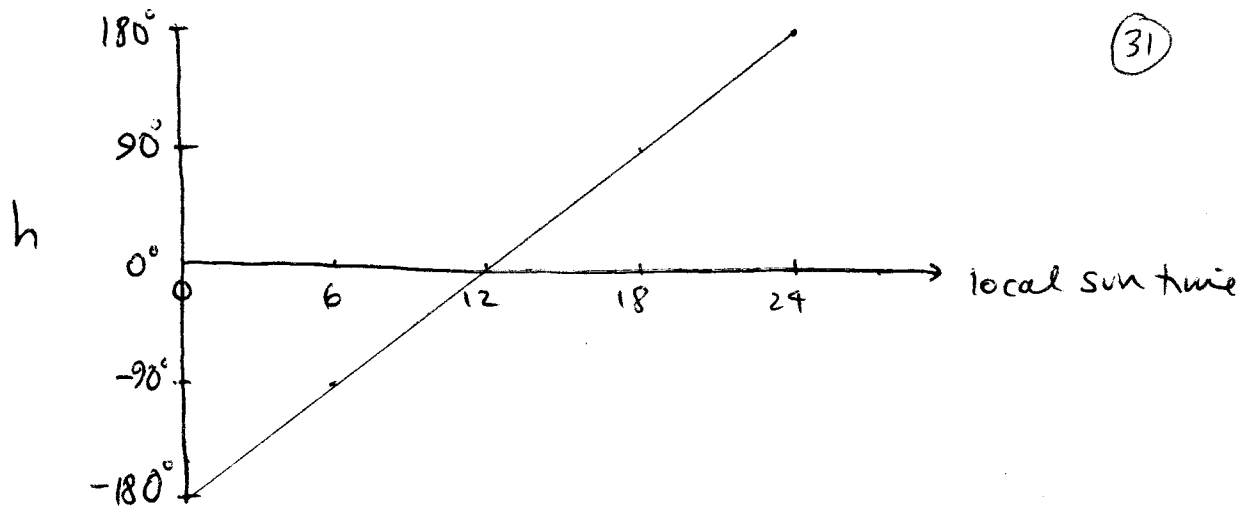
21 June $\delta = +23.45^\circ$

21 March, 23 Sep $\delta = 0^\circ$

22 Dec $\delta = -23.45^\circ$



④ h = hour-angle: the angle through which the earth must turn to bring our longitude directly under the sun. (-180° to $+180^\circ$, negative in morning).
 h is zero at local noon and increases 15° every hour after noon.



so
$$h = 15(t - 12)$$

where h is hour-angle in degrees
and t is local sun time in hours.

or
$$h = \frac{2\pi}{24}(t - 12)$$
 where h is in radians

⑤ with these definitions, Hartmann (Appendix A.1) shows that

$$\cos \theta_s = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \quad (1)$$

B. Time of sunrise and sunset

(32)

Now define H = half-day length, in degrees.

$$\text{so } h = \begin{cases} +H & \text{at sunset} \\ -H & \text{at sunrise} \end{cases}$$

At sunrise & sunset, $\theta_s = 90^\circ$; $\cos \theta_s = 0$

Put $\cos \theta_s = 0$ in (1) and solve for H :

$$\boxed{\cos H = -\tan \phi \tan \delta} \quad (2)$$

C. Instantaneous flux of solar radiation at TOA , F (Wm^{-2})

$$F = S_0 \left(\frac{\bar{r}_{es}}{r_{es}} \right)^2 \cos \theta_s \quad (3)$$

where S_0 = solar constant for mean earth-sun distance = 1370 Wm^{-2}

r_{es} = earth-sun distance

$\bar{r}_{es}$ = mean earth-sun distance

θ_s = solar zenith angle.

D. Total radiation received over the course of a day, E ($J m^{-2}$)

$$E = \int_{\text{sunrise}}^{\text{sunset}} F dt \quad \text{where } t = \text{time (hours)}$$

Change variables from t to h .

$$\frac{dh}{dt} = \Omega, \text{ the rotation rate.}$$

$$\begin{aligned} \Omega &= \frac{2\pi}{24} \text{ radians per hour} = 15 \text{ degrees per hour} \\ &= 2\pi/86400 \text{ radians per second,} \\ &\quad [360 \text{ degrees} = 2\pi \text{ radians}] \end{aligned}$$

$$\begin{aligned} \text{so } E &= \frac{1}{\Omega} \int_{-H}^H F dh \\ &= \frac{1}{\Omega} S_0 \left(\frac{\bar{r}_{es}}{r_{es}} \right)^2 \int_{-H}^H [\sin \phi \sin \delta + \cos \phi \cos \delta \cosh] dh \end{aligned}$$

$$E = \frac{2}{\Omega} S_0 \left(\frac{\bar{r}_{es}}{r_{es}} \right)^2 [H \sin \phi \sin \delta + \sin H \cos \phi \cos \delta] \quad (4)$$

where $\begin{cases} E \text{ is in } J/m^2 \\ H \text{ is in radians} \end{cases}$; Ω is in radians per second

E. Average solar flux over the course of a day (24 hours or 86400 sec)

$$\bar{F} = \frac{E}{86400} \quad W m^{-2}$$

$$\bar{F} = \frac{S_0}{\pi} \left(\frac{\bar{r}_{es}}{r_{es}} \right)^2 [H \sin \phi \sin \delta + \sin H \cos \phi \cos \delta] \quad (5)$$

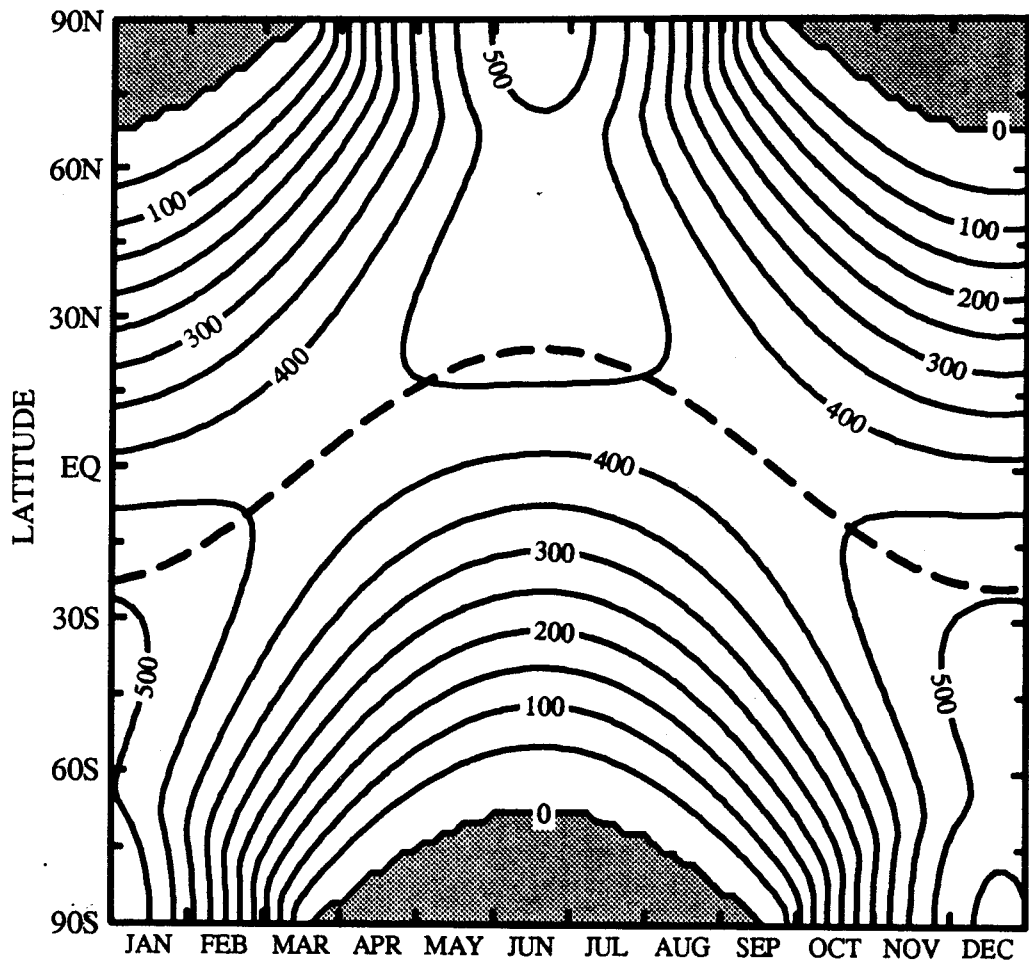


Fig. 2.6 Contour graph of the daily average insolation at the top of the atmosphere as a function of season and latitude. The contour interval is 50 W m⁻². The heavy dashed line indicates the latitude of the subsolar point at noon.

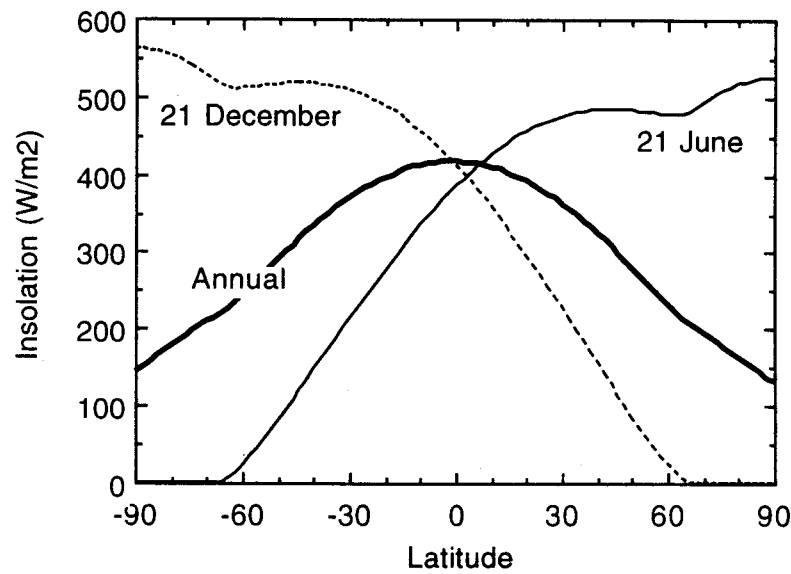
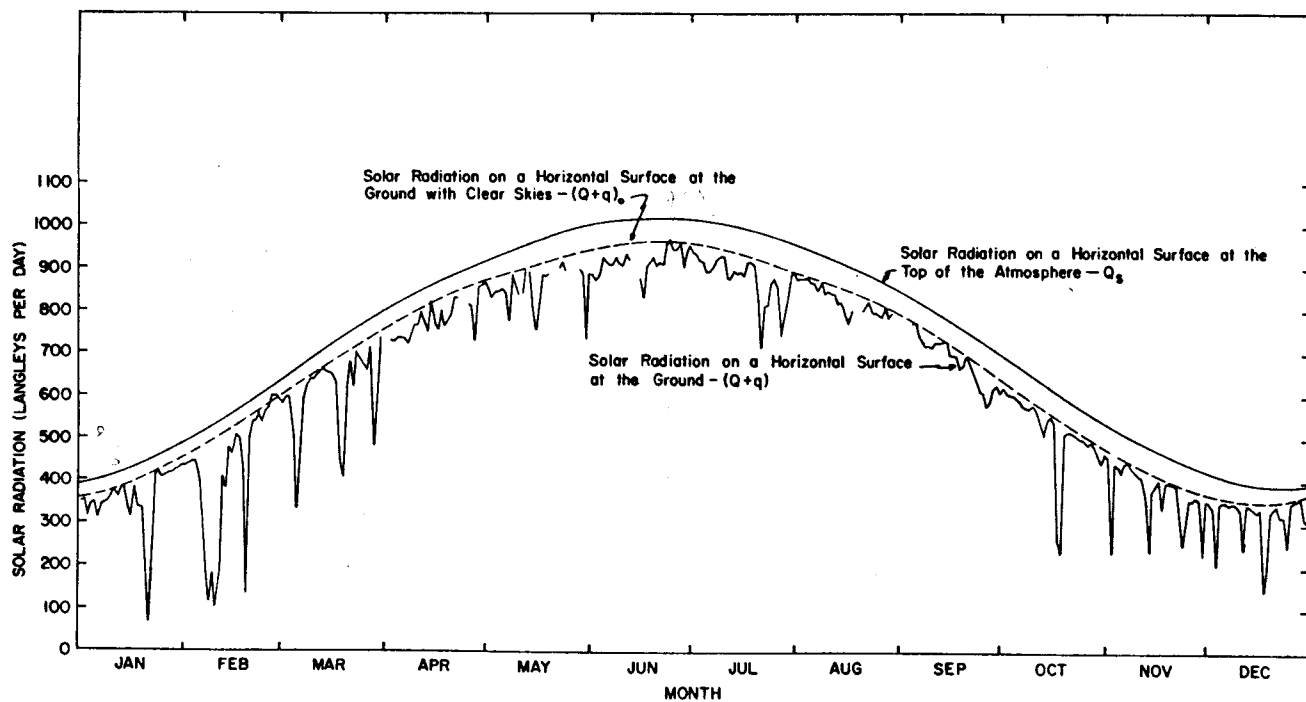


Fig. 2.7 Annual-mean and solstice insolation as functions of latitude.



SELLERS 1965.

FIG. 11.—Solar radiation on a horizontal surface at the top of the atmosphere, Q_0 , at the ground, $Q + q$, and at the ground with clear skies, $(Q + q)_0$, at Inyokern, California ($35^{\circ}39'N$, $117^{\circ}49'W$), during 1962.

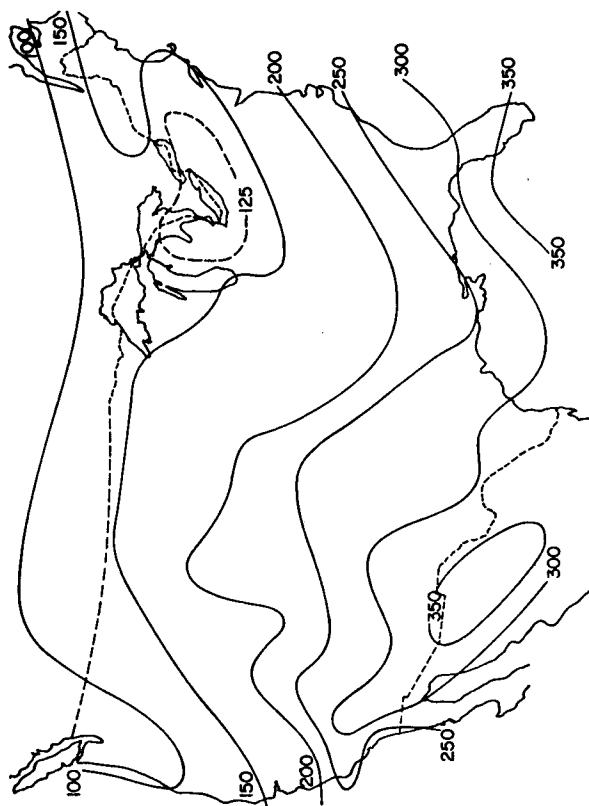


FIG. 9.—The average solar radiation on a horizontal surface at the ground in January in the United States. The units are langley's per day.

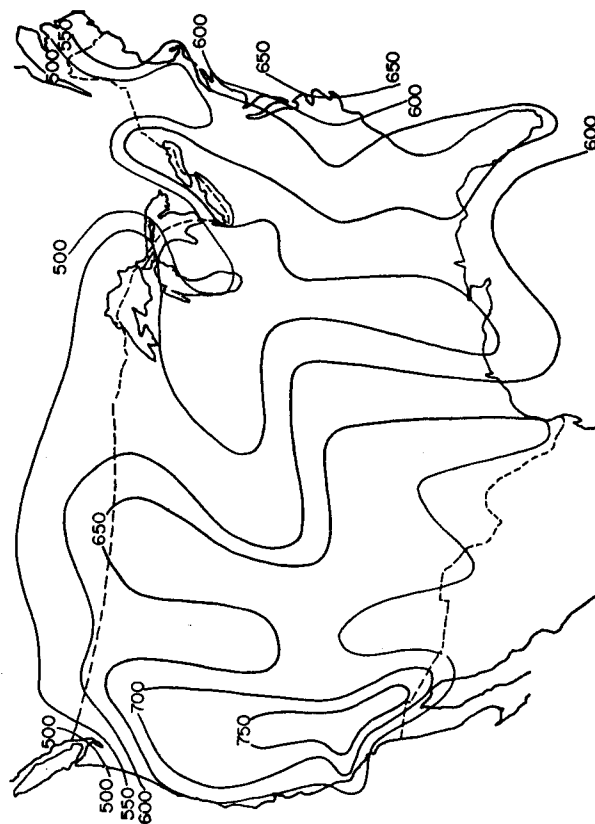


FIG. 10.—The average solar radiation on a horizontal surface at the ground in July in the United States. The units are langley's per day.

1 24/day = 0.484 Wm⁻² SELLERS 1965