Seasonal Simulation as a Test for Uncertainties in the Parameterizations of a Budyko-Sellers Zonal Climate Model

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ABSTRACT

The energy-transport parameterization of Budyko (1969), which was devised to parameterize mean annual net radiation as a function of zonally averaged surface temperature, is subjected to verification with seasonal transport data in order to evaluate its validity for climatic change experiments. It is found that Budyko's linear parameterization is able to describe the annual zonal heat transport divergence for all latitudes and also the seasonal cycle of heat transport divergence at high latitudes ($\phi > 50°$), but has no predictive ability for the seasonal deviation from annual average in lower latitudes.

The parameterization of infrared flux at the top of the atmosphere as a linear function of zonal surface temperature is tested using seasonal data for latitude zones in which the seasonal cycle of temperature has a large amplitude. The temperature coefficients for the different zones examined are found to differ from each other by as much as a factor of 2.

This uncertainty, together with the uncertainty in the strength of the ice-albedo-temperature positive feedback, propagates to an uncertainty in the sensitivity of model global climate to changes in the solar constant. The reduction in solar output required by a simple climate model to generate an ice-covered earth falls roughly in the range of 2 to 21% because of uncertainties in these two radiative coefficients alone. Uncertainty in the transport parameterization would further increase this range.

1. Introduction

Zonally averaged, annually averaged, energy-balance climate models are oversimplified in some respects, but can be useful in isolating the magnitudes of temperature change produced by various agents of climatic change. Their advantage lies in their extreme simplicity: the computation required is minimal (some versions can be solved analytically) and physical cause-and-effect relationships are readily understood.

The prototypical model (Budyko, 1969) is a set of equations, one for each latitude band, of the form

$$F_{\downarrow} - F_{\uparrow} = F_{\leftrightarrow}, \qquad (1)$$

where $F_{\downarrow}$ is the incoming solar radiation, $F_{\uparrow}$ the outgoing infrared radiation, and $F_{\leftrightarrow}$ the divergence of energy transport by ocean and atmosphere out of the latitude zone (henceforth termed "net transport"). These equations have no time dependence. They assume no net storage of energy and therefore are valid only for the yearly average of an equilibrium climate.

The approach is to express each term in (1) as a function of zonally averaged surface air temperature, although certain embellishments on this procedure are possible (such as division of each latitude zone into a land and a sea part). Most modelers have adopted Budyko's original suggestion of approximating the outgoing IR flux as a linear function of surface temperature, especially since such an approximation seems in remarkable concord with satellite data (Cess, 1976). There has been substantially less agreement about the solar and transport terms. Alternative parameterizations for earth-atmosphere albedo (Budyko, 1969; Sellers, 1969) and for transport (Budyko, 1969; North, 1975a,b; Gal-Chen and Schneider, 1976) have been tried. Lindzen and Farrell (1977), for example, recently suggested some modifications to the transport term that could be incorporated while still retaining the model's essential simplicity.

The particular form of (1) adopted by Budyko was

$$QS(\phi)[1 - \alpha(\phi)] - [A + BT(\phi)] = C[T(\phi) - \bar{T}], \qquad (2)$$

where Q is one-fourth of the solar constant. The present value of Q is taken to be 343 ± 5 W m^{-2} (Fröhlich, 1977). $S(\phi)$, the normalized distribution function for insolation, has been tabulated by Chýlek

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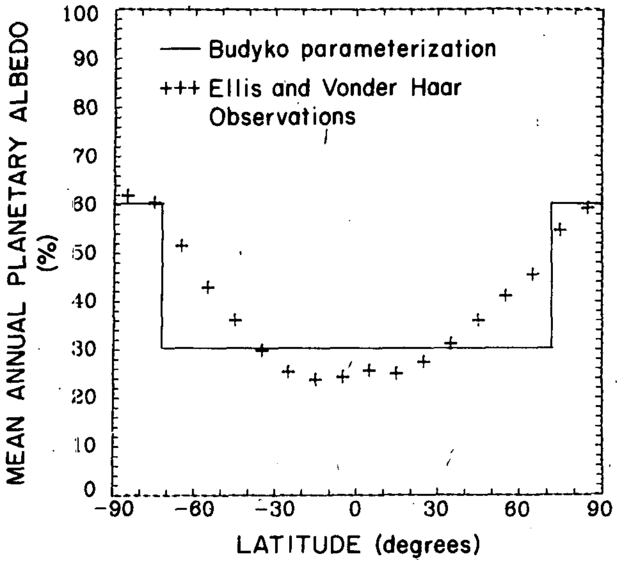


FIG. 1. Annual mean planetary albedo. Plus marks indicate present observed values (Ellis and Vonder Haar, 1976). Solid line: step-function albedo with ice-line at $\phi = 72°$. Budyko (1969) used $\alpha_{ice} = 0.62$, $\alpha_{ice-free} = 0.32$. Lindzen and Farrell (1977) used $\alpha_{ice} = 0.60$; $\alpha_{ice-free} = 0.30$ to agree better with the updated value of global average albedo of 0.303 [computed from data of Ellis and Vonder Haar (1976)]. The latter values are plotted here.

and Coakley (1975). $\alpha(\phi)$ is the planetary albedo. Budyko expressed it as a stepfunction

$$\alpha = \begin{cases} \alpha_{ice-free}, & |\phi| < \phi_{ice} \\ \alpha_{ice}, & |\phi| > \phi_{ice}, \end{cases} \tag{3}$$

where ϕ_{ice} is the latitude of the mean ice line. In order to match approximately the present estimate for global albedo of 0.303 [which we calculate from the data of Ellis and Vonder Haar (1976)], these terms for the Budyko model are taken as

$$\left.\begin{aligned} \alpha_{ice-free} &= 0.30 \\ \alpha_{ice} &= 0.60 \\ \phi_{ice} &= 72° \end{aligned}\right\} .$$

Fig. 1 compares this albedo function with the observed zonal albedo distribution given by Ellis and Vonder Haar (1976) and is discussed further in Section 3a.

Sellers (1969) introduced an alternative expression for albedo, which we use in Section 3 of this paper:

$$\alpha(\phi) = \begin{cases} b(\phi) - f[T(\phi) + 273.15], & T < 10°\mathrm{C} \\ b(\phi) - f[10 + 273.15], & T > 10°\mathrm{C}, \end{cases} \tag{4}$$

subject to the limitation that α cannot exceed 0.85. In this formulation, the albedo is a function of temperature only when the mean annual zonal temperature is low enough (<10°C) for snow or sea-ice cover to be expected for at least part of the year at some longitudes within the zone. Because of the nonuniform land-sea distribution, the snow line does not follow a latitude line. Sellers' formula accounts for this by allowing the zonal albedo to vary smoothly, rather than abruptly, with latitude. For any given value of f, the temperature-independent components $b(\phi)$ are "tuned" to give the present observed annual values of planetary albedo; f is the "albedo-temperature feedback coefficient," which Sellers (1969) thought most likely to be 0.009 K^{-1}.

The outgoing infrared flux at the top of the atmosphere in (2) is expressed as a linear function of T,

$$F_{\uparrow}(\phi) = A + BT(\phi). \tag{5}$$

Budyko's original suggestion assumed that the IR flux was also linearly dependent on the cloudiness fraction and on the product of cloudiness fraction and surface temperature, but cloudiness was then assumed constant, so the equation could be reduced to (5). This expression is discussed in Section 3b.

The term for net energy transport in (2) is also a linear function of T,

$$F_{\leftrightarrow}(\phi) = C[T(\phi) - \bar{T}], \tag{6}$$

where $\bar{T}$ is the global annual mean surface air temperature. Eqs. (5) and (6) were suggested by Budyko (1969). Eq. (6) is a fit to observed temperatures and to calculated net radiation, and is not meant to represent an individual physical transport process.

This set of equations (2), one for each latitude belt, is suitable for investigating factors that have a mean annual influence, e.g., a change in the solar constant Q, a change in the insolation distribution $S(\phi)$ due to changed obliquity of the earth's axis (Suarez and Held, 1976), or a change in the infrared coefficients A and B due to changed chemical composition of the atmosphere. However, any such mean annual model cannot be used to investigate an external forcing which operates by virtue of its interaction with the seasonal cycle.

Each of the three terms in (2) has a coefficient which describes its dependence on surface temperature:

$$\frac{1}{QS(\phi)}\frac{\partial F_{\downarrow}}{\partial T} = \begin{cases} f, & T < 10°\mathrm{C} \\ 0, & T > 10°\mathrm{C}, \end{cases}$$

$$\frac{\partial F_{\uparrow}}{\partial T} = B,$$

$$\frac{\partial F_{\leftrightarrow}}{\partial T} = C.$$

There is uncertainty about the values of f and B; the effects of these parameters on the graph of ϕ_{ice} vs Q are considered in Section 4. The value of

TABLE 1a. Monthly mean zonally averaged surface air temperatures* (°C) at latitude intervals of 5°.

Latitude	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual average
90°N	−36.4	−32.5	−31.6	−25.5	−12.7	−2.4	0.6	−0.4	−10.6	−20.4	−28.2	−30.9	−19.2
85	−33.9	−31.8	−30.9	−24.6	−12.1	−1.8	1.5	−0.1	−9.3	−18.3	−25.5	−29.4	−18.0
80	−28.5	−28.6	−28.9	−22.1	−10.3	−2.0	1.6	0.1	−5.5	−15.3	−22.4	−27.2	−15.8
75	−24.8	−25.4	−24.5	−18.5	−8.4	−0.6	2.8	1.4	−2.6	−10.4	−19.2	−22.7	−12.7
70	−23.8	−24.6	−21.3	−14.2	−4.8	2.8	7.0	5.7	0.9	−7.7	−16.8	−22.0	−9.9
65	−22.6	−20.7	−15.9	−7.0	2.1	8.7	12.3	10.4	4.9	−3.9	−14.1	−20.3	−5.5
60	−15.4	−14.3	−9.3	−1.8	4.7	10.2	13.3	12.2	7.7	0.9	−7.3	−13.2	−1.0
55	−10.6	−9.5	−5.3	1.5	7.2	11.7	14.5	13.8	10.1	4.1	−3.0	−8.3	2.2
50	−7.5	−6.1	−2.2	4.2	9.4	13.3	16.1	15.8	12.1	6.6	0.1	−5.3	4.7
45	−2.4	−1.0	2.0	7.3	11.7	15.5	18.4	18.7	15.5	10.4	4.3	0.0	8.4
40	3.7	4.8	7.3	11.5	15.0	18.7	22.0	21.8	19.2	14.4	9.3	5.5	12.8
35	8.2	8.9	11.1	14.3	17.3	21.0	23.5	23.8	21.7	17.7	13.2	10.0	15.9
30	13.1	13.7	15.9	18.6	21.3	24.1	25.8	25.7	24.6	21.7	17.9	14.9	19.8
25	18.0	18.5	20.7	22.7	24.8	26.6	27.2	27.3	26.8	24.7	22.1	19.4	23.2
20	21.3	21.8	23.4	24.9	26.6	27.5	27.6	27.8	27.5	26.2	24.3	22.3	25.1
15	24.0	24.5	25.6	26.6	27.7	27.6	27.3	27.1	27.3	27.0	25.9	24.6	26.3
10	25.5	25.9	26.6	27.2	27.5	27.2	26.9	26.7	26.8	26.7	26.4	25.7	26.6
5	26.0	26.5	26.7	27.0	26.8	26.5	26.1	26.0	26.1	26.2	26.2	26.1	26.3
0	26.1	26.6	26.8	26.9	26.7	26.3	25.6	25.6	25.9	25.9	26.1	26.1	26.2
5	26.4	26.8	27.0	27.1	26.7	26.0	25.4	25.3	25.6	25.9	26.2	26.2	26.2
10	26.3	26.5	26.7	26.6	26.1	25.2	24.5	24.4	24.8	25.3	25.6	25.9	25.7
15	25.8	26.0	26.1	25.6	24.6	23.4	22.7	22.9	23.7	24.3	24.9	25.4	24.6
20	25.5	25.7	25.5	24.4	22.7	21.3	20.7	20.9	21.8	22.9	23.8	24.8	23.3
25	24.6	24.8	24.2	22.5	20.6	18.9	18.0	18.4	19.2	20.5	22.1	23.4	21.4
30	22.7	23.0	22.3	20.5	18.4	16.7	15.9	15.9	16.7	18.0	19.7	21.3	19.3
35	19.6	20.1	19.5	17.8	16.1	14.5	13.8	13.6	14.0	15.0	16.6	18.2	16.6
40	16.1	16.5	16.0	14.5	13.0	11.7	11.0	10.8	11.1	11.9	13.4	14.8	13.4
45	11.9	12.3	11.9	10.5	9.4	8.3	7.6	7.4	7.8	8.6	9.7	10.8	9.7
50	8.0	8.4	8.0	6.8	5.7	4.8	4.4	4.2	4.5	5.2	6.3	7.1	6.1
55	5.1	5.2	4.7	3.4	2.2	1.3	0.9	0.7	1.1	1.9	3.2	4.1	2.8
60	2.5	2.4	1.6	−0.1	−1.7	−3.1	−4.2	−4.3	−3.6	−1.9	0.1	1.7	−0.9
65	0.1	−0.6	−2.4	−5.3	−7.7	−9.6	−11.6	−12.0	−11.0	−7.4	−3.7	−0.6	−6.0
70	−6.5	−10.1	−14.0	−18.9	−22.0	−23.8	−26.4	−26.2	−25.3	−20.7	−13.1	−7.3	−17.9
75	−17.6	−23.8	−32.6	−35.8	−41.1	−41.5	−42.4	−43.6	−42.4	−36.1	−25.1	−17.8	−33.3
80	−22.2	−29.8	−43.2	−45.4	−49.8	−49.5	−49.7	−52.2	−50.5	−43.0	−31.5	−23.0	−40.8
85	−26.1	−33.4	−48.1	−50.7	−52.2	−51.0	−51.6	−54.1	−53.3	−45.2	−33.2	−26.4	−43.8
90°S	−28.4	−38.6	−53.1	−58.1	−57.6	−58.3	−58.3	−58.7	−57.4	−51.0	−39.5	−28.8	−49.0

* Monthly zonal surface temperatures for the Southern Hemisphere are given by Taljaard *et al.* (1969). Monthly surface temperatures for the Northern Hemisphere are given by Crutcher and Meserve (1970); zonal averages were compiled from these data for the Northern Hemisphere by Roy Jenne and supplied to us by H. van Loon. The annual values listed here have been compiled by us from the zonal monthly averages.

C which Budyko found to fit the present climate is 3.74 W m^{-2} K^{-1}. Held and Suarez (1974, their Fig. 3) and Lindzen and Farrell (1977, their Fig. 3) have plotted the effects of different hypothetical heat transport coefficients and showed that the shape of the curve ϕ_{ice} vs Q is sensitive to the value of C, and that the "global stability" is greatly increased if the efficiency of meridional heat transport is reduced. Within the context of the model, the value of global stability depends on the values of f, B and C, but implicit in this analysis is that the functional forms of the parameterizations themselves would be valid for the range of climatic changes of interest.

Here it is important to clarify the terms "sensitivity" and "stability." We may define an "ice-edge sensitivity" to small perturbations in the solar constant for the present climate (at Q near Q_0): $\partial\phi_{ice}/\partial Q\,|_{Q\approx Q_0}$. This is a sensitivity to only one variable, the solar flux. "Global stability," on the other hand, is defined by Lindzen and Farrell (1977) as the fraction by which the solar constant must decrease in order to lead to a completely ice-covered earth. The latter expresses the ability of the earth to resist total glaciation in the face of a large reduction in solar luminosity.

The reason that "global stability" was found to be higher for smaller C is that an albedo change in one latitude zone has a smaller impact on the temperatures in neighboring zones if the heat transport is more sluggish. In the extreme case of zero transport, the albedo-temperature feedback operates within a latitude zone but does not affect the energy balance of neighboring zones. A larger global stability usually means a smaller local sensitivity.

In this paper we test parameterizations for

TABLE 1b. Annual data for 10° latitude zones.

Zone	$T(\phi)$, annual mean[a] (°C)	$S(\phi)$[b]	$\alpha(\phi)$[c]	$F_{\leftrightarrow}$,[c] annual mean, (W m^{-2})
80–90°N	−16.9	0.500	0.589	−103
70–80	−12.3	0.531	0.544	−94
60–70	−5.1	0.624	0.452	−72
50–60	2.2	0.770	0.407	−47
40–50	8.8	0.892	0.357	−21
30–40	16.2	1.021	0.309	1
20–30	22.9	1.120	0.272	18
10–20	26.1	1.189	0.248	46
0–10	26.4	1.219	0.254	59
0–10	26.1	1.219	0.241	56
10–20	24.6	1.189	0.236	41
20–30	21.4	1.120	0.251	22
30–40	16.5	1.021	0.296	0
40–50	9.9	0.892	0.358	−27
50–60	2.9	0.770	0.426	−57
60–70	−6.9	0.624	0.513	−86
70–80	−29.5	0.531	0.602	−90
80–90°S	−42.3	0.500	0.617	−88

[a] Surface temperatures obtained by area-weighted, three-point, binomial smoothing of the data for 5° zones in Table 1a.
[b] Insolation function from Chýlek and Coakley (1975).
[c] Planetary albedos and net heat transports from Ellis and Vonder Haar (1976).

transport and IR flux individually against observations of transport and IR flux, both annually and monthly. The Budyko transport parameterization is found to generate adequately at all latitudes the annual zonal values (to which it is "tuned") but is a poor predictor of seasonal deviations from mean annual values in the tropical zones.

We find that the temperature dependences of albedo and IR flux both appear to be uncertain over a range of a factor of 2. Thus, even assuming no error in the Budyko transport parameterization, it is plausible that a decrease in solar constant as large as 21% might be necessary for complete glaciation of the entire earth.

Table 1 lists the data used in this paper.

2. How well does the Budyko parameterization predict heat transport?

We would like to analyze Budyko's heat transport expression by the use of a direct and independent test. By "direct" we mean to evaluate its prediction of net heat transport, rather than its prediction of indirect effects (e.g., temperature). For this direct test we do not require the whole model; we use only the transport term (6). By "independent" we mean to test the parameterization with data to which it was not tuned. Budyko derived his transport formula from mean annual zonal data, so an independent test would be its prediction of seasonal variations. We first compare parameterization-generated net transport with observed net transport for annual values (i.e., a direct but not independent test) and then proceed to a seasonal test. Of course, an annual value is the sum or average of the seasonal values, so seasonal values are not strictly independent of the annual. But the seasonal *deviations* from the annual mean are independent of the mean, so testing against seasonal deviations is an independent test.

a. Mean annual energy transport

1) BUDYKO MODEL

We assume there is no *net* heat flux into the oceans and cryosphere from one year to the next. Then, on a mean annual basis, the net radiation excess at the top of the atmosphere in each latitude zone is balanced by heat transport out of the zone, according to (1). In Fig. 2a we have plotted the net radiation ($F_{\downarrow} - F_{\uparrow}$) taken from satellite data by Ellis and Vonder Haar (1976), together with the predicted net heat transport we calculate with the linear Budyko transport parameterization using *observed* surface temperatures. We see that the linear parameterization works rather well (mean deviation 9.7 W m^{-2}) except in the latitude zones 70–80°S and 80–90°S. Here the surface elevations are high relative to all other zones and there is also typically an extreme temperature inversion at the surface (Schwerdtfeger, 1970, p. 275), both of which suggest that the surface temperature is not the same as some effective temperature for transport (assuming that such a parameterization simply in terms of temperature is possible in these latitudes).

2) LINDZEN AND FARRELL MODEL

Lindzen and Farrell (1977, hereafter LF) suggested two modifications to the Budyko heat transport parameterization. [The justification for these modifications is criticized by Warren and Schneider (1979).] The first modification was to set $S(\phi)$ constant at its mean value over the range $-25° < \phi < 25°$, in order to mimic the hypothesized temperature-smoothing effect of a Hadley cell. The difference between the true insolation and the adjusted insolation is an effective transport operating among the zones $-25° < \phi < 25°$. LF further separated "atmospheric eddy" transport (transport coefficient C_A) acting between the pole and some latitude in the tropics, and "oceanic" transport (transport coefficient C_0) acting between the equator and the ice line. Their best guess was that $C_A \approx C_0$, and that the atmospheric eddy transport began somewhere between 10° and 20° latitude. [We use 15° latitude in our calculations with the LF model.] C_0 is tuned such that $T(\phi_{ice}) = -6°C$ on the equatorward side of the ice boundary. In order

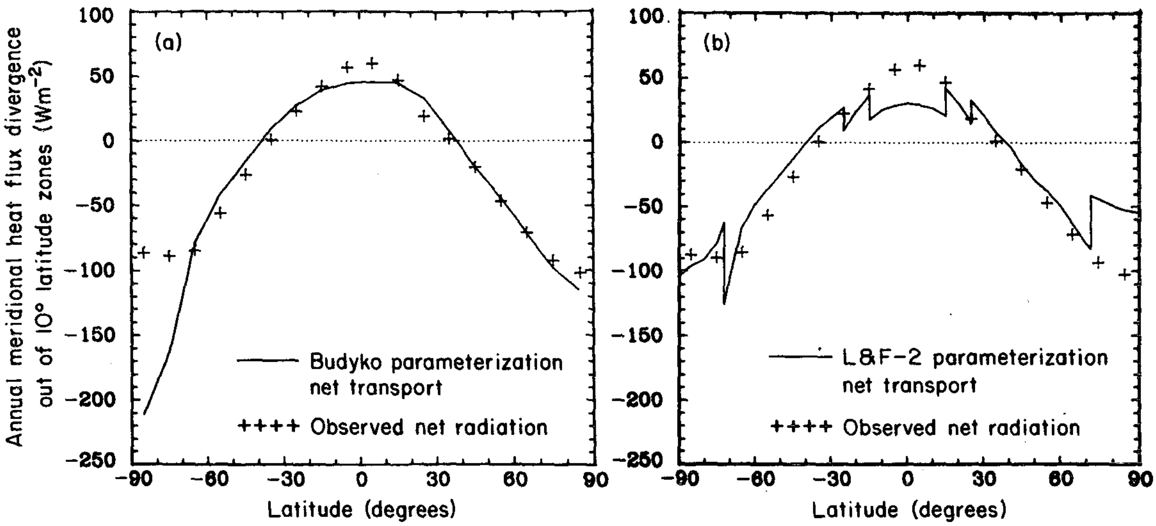


FIG. 2. Test of heat transport parameterizations, for annual mean net transport. Plus marks indicate observed values of net radiation, $F_\downarrow - F_\uparrow$, from Ellis and Vonder Haar (1976).
(a) Solid line: Net energy transport predicted by Budyko's linear formula (6), with $C = 3.74$ W m^{-2} K^{-1}. $T(\phi)$ are the *observed* annual mean temperatures, and $\bar{T}$ is the global annual mean temperature (14.22°C). Calculations were done at 10° latitude intervals, and the line is a linear interpolation between calculated values. The area-weighted mean deviation from observations is 9.7 W m^{-2}.
(b) Solid line: Net energy transport predicted by transport parameterization of model LF-2, from *observed* annual mean temperatures. This prediction includes both the explicitly-modeled transport and an equivalent "transport" due to the Hadley adjustment [Eq. (10)]. The value of C for use in (7) is obtained as follows: A is first chosen to give *global* energy balance with the observed $\bar{T}$ (14.2°C); no transport term required. Then C_0 is chosen so as to set $T = -6$°C on the equatorward edge of the ice line. C is obtained from (8) and used with observed values of $T(\phi)$ and $\tilde{T}$ to get the predicted transport. $\tilde{T} = 13.5$°C (from (9)).
Calculations were done at 10° latitude intervals and additionally at the discontinuities at 72°, 25° and 15°. The line is a linear interpolation between calculated values. The area-weighted mean deviation from observations is 14.7 W m^{-2}.

to ensure energy balance, the effective global temperature ($\tilde{T}$) for use in the transport formula is different from $\bar{T}$ and is obtained as a weighted average. The transport formulation for the present climate is thus

$$F_{\leftrightarrow}(\phi) = C(\phi)[T(\phi) - \tilde{T}], \tag{7}$$

where

$$C(\phi) = \begin{cases} C_0, & |\phi| < 15^\circ \text{ or } |\phi| > 72^\circ \\ 2C_0, & 15^\circ < |\phi| < 72^\circ \end{cases} \tag{8}$$

$$\tilde{T} = \frac{\int_0^{\pi/2} T(\phi)C(\phi)\cos\phi d\phi}{\int_0^{\pi/2} C(\phi)\cos\phi d\phi}. \tag{9}$$

The value $2C_0$ in Eq. (8) for midlatitudes arises because $C_A = C_0$:

$$C(\phi) = C_A + C_0 = 2C_0.$$

We label this model LF-2. For comparison with observed net transports, we must add the explicitly modeled net transport and the effective net transport due to the Hadley adjustment:

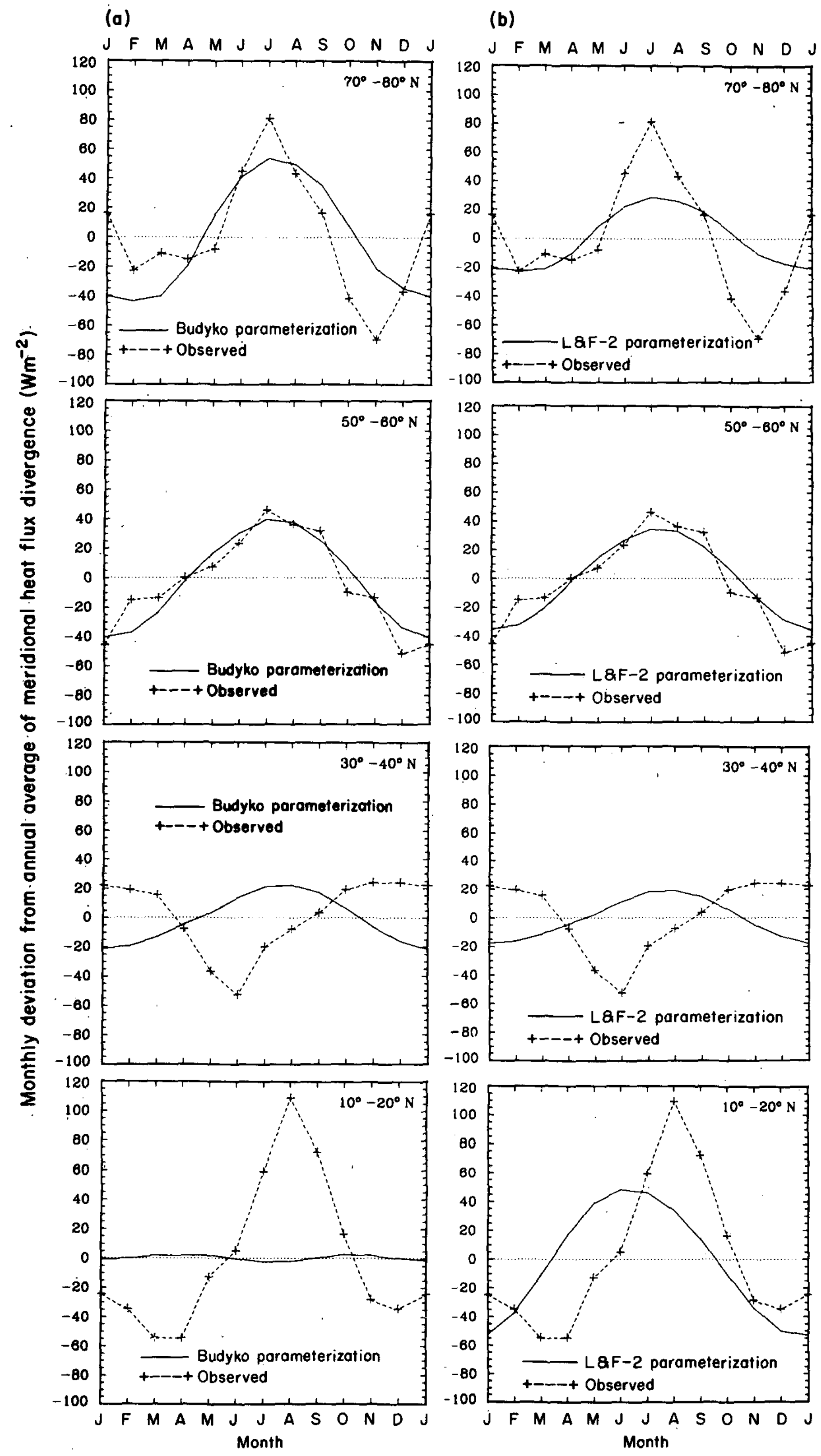
$$F'_{\leftrightarrow}(\phi) = C(\phi)[T(\phi) - \tilde{T}] + Q(\Delta S)[1 - \alpha(\phi)]. \tag{10}$$

ΔS is the difference between $S(\phi)$ and its average over the domain $-25° < \phi < 25°$; $\Delta S = 0$ outside this domain.

Fig. 2b shows the net heat transport predicted by (10) for present observed zonal temperatures. This parameterization, which halves the transport coefficient in the tropics and in the polar regions, is seen to improve agreement with observations over Antarctica and to worsen agreement over the Arctic Ocean. The area-weighted mean deviation from observations is 14.7 W m^{-2}. The LF-2 parameterization underestimates the net heat export from the tropics worse than does the Budyko parameterization. With the partial decoupling of the tropics and ice caps from the midlatitudes which is implied by this modification, a greater reduction in the solar constant was found necessary to glaciate the tropics (Lindzen and Farrell, 1977).

b. Seasonal energy transport

Although Budyko's transport parameterization works rather well for annual mean values of the present climates, the fit could be fortuitous (i.e., a curve fit) and therefore not valid for climatic change applications. Since the linear transport formulation does not model an individual physical process,


(a)
(b)
J F M A M J J A S O N D J
70° -80° N
50° -60° N
30° -40° N
10° -20° N
Budyko parameterization
Observed
L&F-2 parameterization
Monthly deviation from annual average of meridional heat flux divergence (Wm^-2)
Month

we have less confidence that it would be valid for a changed climate, which the model is designed to predict. We have no clear physical justification for using it, so we must give it a stringent empirical test.

A model which is tuned to the present annual climate can best be verified or invalidated by observing the accuracy of its predictions for documented climatic changes. Here we use the seasons as an example of such climatic change.

For the seasons (1) becomes, for each latitude zone,

$$F_{\downarrow}(\phi) - F_{\uparrow}(\phi) = F_{\leftrightarrow}(\phi) + \mathrm{St}(\phi),$$

where St is the rate of heat storage. This storage is primarily into (and out of) the ocean, except at high latitudes where the cryosphere also contributes.

The atmospheric and oceanic components of the net transport, as well as $\mathrm{St}(\phi)$, have been extracted from satellite and near-surface data for the Northern Hemisphere by Oort and Vonder Haar (1976). Direct observations of oceanic storage and atmospheric transports allow the calculation of oceanic transport as a residual. Thus, values of net oceanic transport are accurate on the average to probably no better than a factor of 2 (Oort and Vonder Haar, 1976, their Tables 8 and 13).

We have plotted the observed seasonal signal of meridional heat flux divergences (atmospheric plus oceanic) for the Northern Hemisphere by latitude band in Fig. 3a, together with those we calculated with the Budyko transport parameterization (6), using for $\bar{T}$ the global *monthly* mean rather than the annual mean. In Fig. 3b the same comparison is done with calculations from the LF-2 transport parameterization (10). [For the latter we used monthly values of the insolation function S in the tropical zones (supplied to us by J. Coakley).] In each case, the annual mean (of the model parameterization or of the observations, respectively) for that latitude zone has been subtracted out, so that only the seasonal signal is plotted. The "seasonal signal," both for the parameterization and for observations, is thus the monthly *deviation* of net transport from the annual average, and its time mean is zero. [To obtain the seasonal net transport, the annual net transport (Fig. 2a) must be added to the seasonal signal. The two discrepancies (between model and data) must likewise be added to obtain the total seasonal discrepancy.]

TABLE 2. Budyko parameterization prediction compared with observed seasonal signal of net transport.

Latitude zone (°N)	Mean observed seasonal signal, $F_{\leftrightarrow}$ (month) − $F_{\leftrightarrow}$ (annual) (W m^{-2})	Difference between parameterization-generated[a] and observed seasonal signal, mean magnitude (W m^{-2})	
		Budyko	LF-2[b]
80–90	38	20	26
70–80	34	25	24
60–70	45	17	20
50–60	25	9	9
40–50	9	19	16
30–40	21	32	30
20–30	10	19	34
10–20	42	43	36
0–10	12	15	17

[a] Predicted net transports are calculated using the transport term alone [Eq. (6) for Budyko or (10) for LF-2], from *observed* temperatures.

[b] Predicted net transport includes both the explicitly modeled transport and an equivalent "transport" due to the Hadley adjustment [Eq. (10)].

The discrepancies between these curves are summarized in Table 2. They can be attributed to 1) inadequacy of the parameterization and 2) observational uncertainties. The Budyko parameterization is seen to work best at high latitudes and worst in the tropics. At high latitudes ($\phi > 50°$) it has considerable predictive ability. At 30–40°N, Oort and Vonder Haar (1976) show net oceanic heat import exceeding net atmospheric heat export for May, June and July, resulting in a net heat flux convergence in these months. The model parameterization predicts for these months a net heat *export* (presumably to the winter hemisphere) which apparently does not occur, so the prediction is opposite to the observed signal. The model's worst prediction is for the zone 10–20°N. In this zone the difference between zonal monthly temperature and global monthly temperature is nearly the same for all months, so *no* seasonal cycle is predicted. We note that the large value of observed net transport

FIG. 3. Seasonal test of heat-transport parameterizations for the Northern Hemisphere, using for $\bar{T}$ or $\tilde{T}$ the global monthly mean. Plotted are the deviations of monthly net transport from the annual average, for representative latitude zones. Observed and calculated values are plotted for each month. The points have been joined by straight lines for display purposes only. Solid line in all cases: difference between monthly and annual values of energy transport predicted by Budyko's linear formula (6), with $C = 3.74$ W m^{-2} K^{-1}. Dashed line: observed monthly deviation of energy transport (oceanic + atmospheric) from Oort and Vonder Haar (1976), excluding storage. Discrepancies are summarized in Table 2b. (a) Budyko parameterization (6). (b) Lindzen and Farrell parameterization (10).

for this zone in July, August and September is *interhemispheric* ocean transport (Oort and Vonder Haar, 1976), i.e., this heat is transported equatorward, not poleward, out of this zone. (If we would use for $\bar{T}$ the hemispheric mean instead of the global mean the model would predict net poleward transport only, and the prediction would still be poor for low latitudes. At 30–40°N the zonal monthly temperature is always close to the hemispheric monthly temperature, so no seasonal cycle would be predicted. At 10–20°N the prediction would be in the opposite direction to the observed signal, because the parameterization predicts poleward net transport. Thus, it would predict larger net transport out of the 10–20°N zone in the winter than in the summer.)

Fig. 3b shows a simulation of the seasonal cycle using the transport parameterization of model LF-2. As one would expect, its prediction is closely similar to that of the Budyko parameterization for 30–70°N. In the Arctic, where the Budyko parameterization simulates the seasonal cycle rather well, the LF-2 parameterization underpredicts the magnitude of the signal. At 10–20°N, we saw that the Budyko parameterization predicts no seasonal cycle. The LF-2 parameterization does have a seasonal cycle, but with the phase advanced about two months. The "Hadley adjustment" is responsible for the seasonal cycle. It causes a larger predicted heat export out of this zone at the time of high sun, because this is the time of greatest difference between $S(\phi)$ and its average. The peak in observed net transport comes two months later, at the time of maximum strength of the *Southern Hemisphere* Hadley cell. However, the net transport out of this zone apparently is due almost entirely to *oceanic* transport [August heat flux divergences, 10–20°N (from Oort and Vonder Haar, 1976) are 158 W m^{-2} oceanic, -4 W m^{-2} atmospheric], but these numbers are uncertain, and a water-mass model of the ocean (Bryan and Lewis, 1979) underpredicts the observed August cross-equatorial heat transport by a factor of 5.

Note again that we have plotted in Fig. 3 only the seasonal signal with annual average subtracted. Since the annual discrepancy for parameterization LF-2 is so great in tropical zones (Fig. 2b), the *total* seasonal net transport values for 0–20°N deviate from observations more than do those of the Budyko parameterization.

Here we have examined only a linear transport parameterization. Thompson and Schneider (1979, their Figs. 3 and 4) have examined the seasonal predictive ability of nonlinear diffusive parameterizations for latent and sensible heat transport in the atmosphere and find that they also are relatively incompetent in the tropics.

We have seen that Budyko's parameterization adequately generates mean annual values of zonal net heat transport. The fact that it fails to describe the seasonal cycle of net transport in the tropical zones causes us to doubt its general utility in climate change experiments. But it is still possible that it would be reliable for predicting *annual averages* in such experiments. This is the assumption we have to make in the model experiments of the next section, where we alter either the infrared formula or the albedo formula, keeping the other parameters fixed.

3. Can we set realistic limits to global stability?

The models of Budyko (1969) and Sellers (1969) require a reduction in the solar constant of only 1.6 to 2% to generate an ice-covered earth. Lindzen and Farrell (1977) suggested that the global stability of a Budyko-type model could be increased by separating the transport into components meant to suggest Hadley circulation, atmospheric eddy transport and transport by ocean currents. In their best guess (their Fig. 15), LF show a decrease of Q by 7% to result in an advance of the ice line to $\phi = 30°$, and a decrease of Q by 17% to be required to result in an ice-covered earth because of the "Hadley stability ledge" at $\phi = 25°$. The appropriateness of their modifications of the Budyko transport formulation to demonstrate the existence of such a "ledge" is questioned by Warren and Schneider (1979).

In addition to the heat flux parameterization, however, we would like to point out, as others have done, other potentially unrealistic features of the Budyko-Sellers model which, if altered, could raise the global stability considerably above 2%. What we examine here are not changes in parameterizations but rather changes in the values of the parameters f and B.

The parameterizations in (2) are derived by developing correlations between observed zonal surface temperatures and observed albedos, IR fluxes and net radiation. Because the parameterizations are oversimplified, different values of the coefficients are obtained for different sets of data. In this section we make estimates of the range of uncertainty for the albedo-temperature coefficient f and the IR temperature coefficient B, and we show the various model-generated plots of ice-line versus solar constant obtained for plausible values of these two coefficients. It is clear from above that the Budyko transport parameterization cannot simulate the seasonal cycle of net heat transport in low latitudes; but since the form of the parameterization itself fails, we have not attempted to estimate a range of possible values for the transport coefficient C. Consideration of an uncertainty in C, of course, would increase the range of values we obtain for global stability. [It is quite possible that a simple regression formula such as (6) cannot explain much of the observed seasonal variability in $F_{\leftrightarrow}$ in tropical zones for any single value of C.]

a. Albedo

Fig. 1 compares observed annual albedo values with those of Budyko's parameterization, in which the albedo jumps from 0.3 to 0.6 at the ice-line but is constant in each of the two domains. The two-mode Legendre approximation of North (1975b) smooths the Budyko stepfunction albedo to resemble the observed $\alpha(\phi)$, and Coakley (1979) has shown that the use of the stepfunction albedo instead of the smoothed albedo does not appreciably change the plot of ϕ_{ice} vs Q. However, Lian and Cess (1977) have argued that the Budyko albedo formulation overestimates the climate sensitivity at $Q = Q_0$ by at least a factor of 2, so that an improved albedo formulation might indicate a greater global stability. We shall instead use Sellers' albedo formulation (4), since it makes albedo an explicit function of temperature, so that the effects of different values of $\partial\alpha/\partial T$ can be investigated. Sellers' formulation produces present observed annual zonal values of albedo (at $Q = Q_0$) for *any* value of the ice-albedo-temperature feedback coefficient f [by tuning the $b(\phi)$]. However, as we shall see below, this formulation is, in some climatic situations, as unrealistic as the stepfunction albedo.

The model can be run either by 1) requiring the present observed $\alpha(\phi)$ to be generated for the present observed $T(\phi)$ or 2) requiring the model to generate the present observed $T(\phi)$ for the present solar constant by adjusting the "present" albedos slightly away from their observed values. We have run all our experiments both ways, and the values of global stability obtained are always nearly identical for both cases. The graphs we present in this paper were calculated in the latter fashion, which means that as f and B are varied, the model still always predicts the present ice-line (72°) for the present solar constant. In this model the Southern Hemisphere is a mirror image of the Northern Hemisphere. Northern Hemisphere observed surface temperatures were used for initialization.

Recall that $-\partial\alpha/\partial T = f$ if $T < 10°\text{C}$ and $\alpha \leqslant 0.85$, and $-\partial\alpha/\partial T = 0$ otherwise. For a given value of $f = -\partial\alpha/\partial T$, values of $b(\phi)$ are chosen so that the model will generate the present distributions of $T(\phi)$ and $\alpha(\phi)$. b is found to vary slightly with ϕ, for example, from about 2.8 to 3.0 if $f = 0.009$.

Sellers (1969) obtained $f = 0.009\ \text{K}^{-1}$ by comparing zonal albedos and temperatures for similar latitudes in the Northern and Southern Hemispheres. For this value of f he obtained a global stability of about 2%. However, he also showed how the use of a hypothetical smaller value, $f = 0.005\ \text{K}^{-1}$, led to an increased global stability of 5%.

Lian and Cess (1977) have recently argued on the basis of satellite data that, although the surface albedo may be a strong function of temperature, the planetary albedo is not, and that Sellers' less favored value of $f = 0.005$ may actually be more realistic than $f = 0.009$. They point out that Sellers' determination of f could have incorporated spurious effects due to the differences between the two hemispheres in land-sea distribution and in cloud amount. Furthermore, the increase in albedo with latitude is not only due to the snow cover but also to the larger zenith angle at high latitudes which raises the cloud albedo (Cess, 1976). Thompson (1979) has shown that the change in cloud albedo due to seasonally changing zenith angle could account for about two-thirds of the amplitude of the *seasonal cycle* in albedo at 50°N; thus apparently only about one-third of the amplitude is due to seasonal variation in snow cover. He has further illustrated that, since there is a seasonal cycle in albedo even in some zones (e.g., 35°S) where the monthly zonal temperature never drops below 10°C, Eq. (4) cannot simulate the seasonal cycle of albedo there for *any* value of f.

Lian and Cess' argument in favor of a smaller f is as follows. From mean annual albedo data (from satellites) and cloudiness data, they have derived values of $\partial\alpha/\partial T$ for each latitude zone in the Northern Hemisphere. These $\partial\alpha/\partial T$ vary strongly with latitude and are applicable only for small changes from the present climate. They are not applicable if the ice-line moves to low latitudes, so one would not use them for the study of "global stability." However, if a single value of $\partial\alpha/\partial T$ is to be chosen for use at all latitudes [to be applied only when $T < 10°\text{C}$ as in (4)], Lian and Cess find that the use of $f = -\partial\alpha/\partial T = 0.004$ gives for the present solar constant the same value of "global climate sensitivity" ($Q_0\ d\bar{T}/dQ$) as does the latitudinally varying $\partial\alpha/\partial T$. We point out below, however, that the use of a constant small value of f may not be justified for a significantly decreased solar constant because it can generate unrealistic albedo values, so we should be skeptical about conclusions based on it.

A further potential source of uncertainty in the albedo-temperature relationship is the uncertainty in the values of the observed albedos (Ellis and Vonder Haar, 1976) used by Lian and Cess to derive their values of $\partial\alpha/\partial T$. Ohring and Adler (1978, their Fig. 6) have plotted the annual average of some new

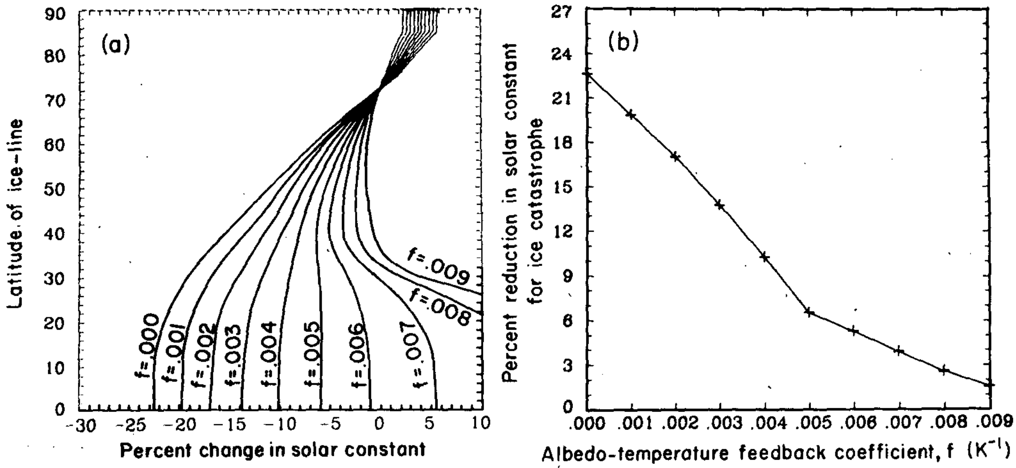


FIG. 4. Effect of changes in the albedo parameter f.

(a) The latitude of the equilibrium ice line is plotted as a function of percent change in solar constant from its present value, for values of the "albedo-temperature feedback coefficient" f, ranging from 0.000 to 0.009 K^{-1}. The Budyko model with Sellers' albedo is used, as described in the text, with $C = 3.74$ and $B = 1.57$ W m^{-2} (°C)$^{-1}$.

The algorithm for obtaining the graph of ϕ_{ice} vs Q is as follows. For the stable branch ($\partial\phi_{ice}/\partial Q > 0$), $T(\phi)$, $\bar{T}$ and $\alpha(\phi)$ were calculated successively and iteration was carried to convergence. For the unstable branch ($\partial\phi_{ice}/\partial Q < 0$), $\bar{T}$ was specified. The value of Q required for energy balance was found by successive trials. The ice line is obtained by inspection as the latitude at which $T = -10$°C.

In this model the Southern Hemisphere is a mirror image of the Northern Hemisphere, and Northern Hemisphere observed surface temperatures were used for initialization. The value of A was chosen such that the model would reproduce the present observed mean annual hemispheric surface temperature; $A = 214.5$ W m^{-2}.

(b) "Global stability" of the Budyko model with Sellers albedo, as a function of the albedo-temperature feedback parameter f, for fixed $C = 3.74$ and $B = 1.57$ W m^{-2} (°C)$^{-1}$. "Global stability" is the percent decrease in Q required for the model to generate an ice-covered earth. This curve is obtained from inspection of the graphs in Fig. 4a.

(unpublished) satellite observations which agreed with those of Ellis and Vonder Haar at low latitudes but were up to 10% higher in the arctic. If there are also discrepancies in the magnitudes of the *seasonal cycles* of zonal albedo, the new observations (when available) might indicate a global value of f different from 0.004 and a correspondingly altered global stability. This discrepancy between albedo observations from different satellites might be attributed to anomalous cloud cover or snow cover during the short sampling times or to differences in channel width. A judgment is difficult to make in the absence of the full set of monthly albedo values from the new satellites. In this paper we have used the albedo values of Ellis and Vonder Haar (1976), which are for 0.2–4.0 μm, i.e., nearly the complete solar spectrum.

The ice-line position as a function of solar constant is plotted in Fig. 4a for several values of f ($f = 0$ means no ice-albedo-temperature feedback, i.e., as Q is decreased all zonal albedos are kept constant at their present values). The "global stability" of these models is taken from these curves and plotted as a function of f in Fig. 4b. The feedback coefficient used by Sellers (1969) was $f = 0.009$, which leads in our version of this model to a global stability of only 1.5%. However, if we use $f = 0.004$, the global stability is increased to 10%. But Lian and Cess (1977) meant the value of $f = 0.004$ to apply only for Q near Q_0 and we shall see that it produces unrealistic consequences if the ice line moves far from its present position.

It is interesting to note that for $f < 0.005$ there is no unstable branch of the curve, so that ϕ_{ice} is a single-valued function[2] of Q. This means that *an ice-covered earth is not a solution of this model for* $Q = Q_0$ *if* $f < 0.005$; therefore, the present climate is "transitive" for small f. [For discussion of transitivity in the context of energy-balance modeling, see Schneider and Gal-Chen (1973).] We have confirmed that the model climate does not exhibit hysteresis when the solar constant is raised stepwise from an ice-covered earth solution. The reason for this is that even when the ice line reaches the equator, the model-produced albedo at the equator is still rather low, e.g., for $f = 0.004$, the equatorial albedo for $T = -10$°C is $\alpha = 0.33$. Because the albedo of clouds increases with solar zenith angle

[2] A study with a similar model (Mokhov and Golitsyn, 1978; Golitsyn and Mokhov, 1978) showed the lack of an unstable branch to require $f \leqslant 0.0037$ K^{-1}.

(Cess, 1976), we might expect the earth-atmosphere albedo over a tropical ice sheet to be less than that over a polar ice sheet, but certainly not as low as 0.33.

However, even with $f = 0.009$, which has a stable ice-covered earth solution at $Q = Q_0$, we get the equatorial albedo for $T = -10°C$ to be 0.43, which is still too low for a planetary albedo over snow. These unrealistic consequences of the albedo parameterization (4) for both $f = 0.009$ and $f = 0.004$ simply point out that the assignment of a single value of $\partial\alpha/\partial T$ for all latitude zones and for all temperatures below 10°C is an oversimplification, as Lian and Cess (1977) also noted. The value of $f = 0.004$ causes the model to mimic Lian and Cess' estimate for the actual climate sensitivity at $Q = Q_0$ (which is of greater practical importance for the present), but our extrapolation becomes less and less believable as we go further from the present climate.

Interestingly, the value of global stability (10%) we obtain for $f = 0.004$ is close to what Coakley (1979) obtained using a more detailed albedo formulation incorporating the recommendations of Lian and Cess, and using a diffusion parameterization for heat transport. He obtained a global stability of about 8.5% under the latter conditions. However, his graph does show an unstable branch, unlike our graph for $f = 0.004$. Since we have seen that the Sellers-type formulation produces unrealistically low albedos for a "tropical ice sheet," we think that the single-valued nature of our curves for $f < 0.005$ could well be an artifact of the model, and we do not take it seriously as evidence that the climate is "transitive" for $Q = Q_0$.

b. Terrestrial infrared radiation

The outgoing IR flux at the top of the atmosphere is assumed in (1) to be a linear function of surface temperature:

$$F_\uparrow = A + BT. \tag{5}$$

Although blackbody radiation is proportional to the fourth power of absolute temperature, the IR opacity of the atmosphere also increases as water vapor content increases with temperature so as to reduce the power to less than 4. This, together with the fact that the range of surface temperatures on earth is small on the absolute scale, is the justification for a linearization. Budyko (1969) used the values $A = 202$ W m^{-2} and $B = 1.45$ W m^{-2} (°C)$^{-1}$. Sellers (1969) used a formula that is more complicated but which is nearly linear over the range $-40°C \leq T \leq 30°C$ so that, in terms of (5), $A = 211$ and $B = 1.68$.

Cess (1976) reevaluated the parameterization using new IR data from satellites (not available in 1969) and cloudiness data, and modified (5) to include a correction for the effect of clouds on IR flux:

$$F_\uparrow = A + BT + kA_c, \tag{11}$$

where k is an empirical constant and A_c the fractional cloud cover. He showed that this simple formula was applicable to zonal annual values for both Northern and Southern Hemispheres. He found the best fit (for the Northern Hemisphere) using zonal annual data to be

$$F_\uparrow = 257 + 1.6T - 91A_c, \tag{12}$$

where the units of $F_\uparrow$ are W m^{-2}. The fit for the Southern Hemisphere was very similar. Some subsequent workers (e.g., Lindzen and Farrell, 1977; Coakley, 1979) have used Cess' formulation (12) but kept fractional cloud cover constant during their climatic change experiments, so in effect they were using (5) with $B = 1.6$. This procedure is followed because it is not clear how cloud cover would change with a change in surface temperature. (Observationally, dA_c/dT is positive for the seasonal cycle in some latitude zones and negative in others.) For discussions of the effect of clouds on IR flux, see Ramanathan (1977) and Cess and Ramanathan (1978).

Our purpose here is to show the level of uncertainty in the value of B for use in (5), and to show the effects on global stability for the range of plausible values. We realize that both water vapor profiles and cloud amounts at different heights are needed for an accurate IR parameterization. However, here we want to examine the uncertainty associated with the use of only a *single* predictor, namely, surface temperature, as has been standard practice in zonal energy-balance models. For use in climate models which do not compute cloudiness, we require the *total* derivative $dF_\uparrow/dT$, implicitly including any change in cloud cover or cloud height with temperature. [Since cloud cover may be a function of latitude as well as of surface temperature, this is not strictly true. But preliminary results of Warren and Thompson (private communication), given below, show that B is nearly the same whether a one-predictor regression $F_\uparrow(T)$ or a two-predictor regression $F_\uparrow(T,A_c)$ is done. Our speculation is that fractional cloud cover is a poor predictor of IR flux because of the large uncertainty in observations of cloud cover.] We have plotted $F_\uparrow$ vs T for various data sets and we get a wide range of values for $B = dF_\uparrow/dT$. These are listed in Table 3 and discussed below. In general, a larger value of B means an enhanced IR-temperature negative feedback, which leads to increased global stability and decreased climate sensitivity (see Appendix of Schneider and Mass, 1975).

In Fig. 5a we plot IR flux versus temperature using mean annual zonal values [temperatures from

Table 1b, IR flux from Ellis and Vonder Haar (1976)]. The best fit for all latitudes is $F_\uparrow = 210 + 1.78T$. But the points fall into two classes. If the data for Antarctica are excluded (70–90°S), a good fit can be drawn through all the other points: $F_\uparrow = 204 + 2.17T$. Oerlemans and van den Dool (1978) obtained $B = 2.23$ as the best fit for all-latitude data, but for 70–90°S they did not use surface temperatures. In order to make T representative of air in the whole column, they ignored two-thirds of the strong inversion over the Antarctic ice cap (Oerlemans, private communication), thus adjusting "surface temperatures" to values much higher than observed, to about −21°C for 70–80°S and −25°C for 80–90°S (cf. Table 1b).

An alternative way to estimate B is to look at the *seasonal* variation of $F_\uparrow$ and T in individual latitude zones. Cess (1976) listed some of these values in his Table 5, based on some preliminary (prepublication) IR data from Ellis and Vonder Haar. We have repeated the plots of $F_\uparrow$ vs T using updated monthly IR data (Ellis and Vonder Haar, 1976) with monthly temperatures (Table 1a). There are only eight latitude zones where the temperature undergoes a seasonal cycle sufficiently large (range > 15 K) for an estimate of $dF_\uparrow/dT$ to be made. These estimates of B range from 1.7 to 3.3 and are listed in Table 3. [We note that for zones 40–50°N and 50–60°N there is little change in fractional cloud cover over the seasonal cycle (London, 1957, his Table 1),[3] so the inclusion of a cloud-cover correction to the IR flux would not be able to reduce these large estimates of B (B = 2.8 and 2.2, respectively).] They

TABLE 3. Estimates of the dependence of outgoing terrestrial radiation on surface temperature.

Reference	$dF_\uparrow/dT$ [W m^{-2} (°C)$^{-1}$]
From annual data, varying latitude	
Budyko (1969)	1.45
Sellers (1969)	1.68
Cess (1976), Northern Hemisphere, fixing A_c	1.57
Oerlemans and van den Dool (1978)	2.23
This work (Fig. 7a), all latitudes	1.78
This work (Fig. 7a), excluding 70–90°S	2.17
From monthly data, fixed latitudes, this work*	
80–90°N	1.72
70–80°N	1.84
60–70°N	1.94
50–60°N	2.18
40–50°N	2.83
30–40°N	3.22
70–80°S	3.35
80–90°S	2.99
Varying latitude and season	
All zones, all months (Fig. 7b)	1.83
Theoretical, from radiative-convective model with constant relative humidity (Ramanathan *et al.*, 1976)	
Fixed cloud-top altitude	2.25
Fixed cloud-top temperature	1.37

* The estimates for 30–90°N are updated values of those made by Cess (1976, his Table 5). Cess' values were based on preliminary (prepublication) IR data of Ellis and Vonder Haar. We have used the published monthly IR data (Ellis and Vonder Haar, 1976).

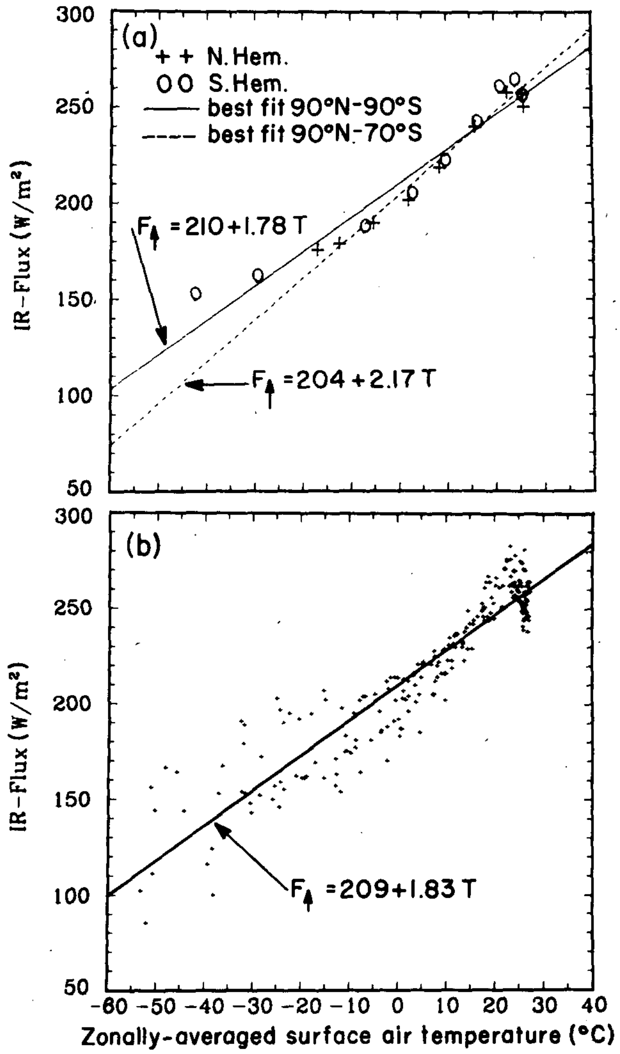


FIG. 5. Outgoing infrared flux as a function of surface temperature for 10° wide latitude zones. IR flux data are from Ellis and Vonder Haar (1976).
(a) Mean annual values. Plus marks indicate Northern Hemisphere; circles, Southern Hemisphere. Solid line: best fit to all points; dashed line: best fit to all points excluding 70–90°S. (b) Monthly values. Points for all 12 months for each of 18 latitude zones are plotted. The line is the least-squares fit to these points.

[3] For 40–50°N and 50–60°N, respectively, the fractional cloud amounts are as follows: Winter 0.59, 0.63; April 0.59, 0.62; Summer 0.55, 0.63; October 0.56, 0.66.

are seen to increase as one moves toward the equator in the Northern Hemisphere. This feature is also predicted by Ramanathan (1977, his Table 2) with the use of a radiative-convective model applied to vertical temperature distributions obtained from GCM results. It is possible that because of the moist convective influences on stabilizing lapse rates in the tropics (e.g., see Fig. 5 of Schneider and Dickinson, 1974), more outgoing IR is radiated to space *per unit surface temperature increase* in lower latitudes than for colder latitudes where moist convection has less influence on lapse rate. This is one of the factors that could account for the larger values of B in low latitudes. But it does not explain the fact that our estimates of B for the Antarctic are also much larger than for the Arctic.

We have also estimated a global value of B by plotting all the monthly values for all the zones (Fig. 5b). There are 216 points in this plot (18 zones × 12 months). The line

$$F_{\uparrow} = 209 + 1.83T \tag{13}$$

is the best least-squares fit, giving a standard error of 15 W m^{-2}. The uncertainty in the observations of $F_{\uparrow}$ is about $\sigma = 5\%$ (Ellis and Vonder Haar, 1976), which averages 12 W m^{-2}. [Preliminary results of Warren and Thompson (private communication) show that a regression on 36 points for the Northern Hemisphere (four months in each of nine zones), *including* cloud amount as a predictor reduced the standard error from 15 to 11 W m^{-2}. The plane of regression was $F_{\uparrow} = 239 + 1.82T - 58A_c$. Comparison with (12) shows that different values of the cloud-cover coefficient k result from using different data sets.]

The uncertainties in the coefficients in (13) are, roughly, $\sigma(A) = 1$ W m^{-2} and $\sigma(B) = 0.05$ W m^{-2} (°C)$^{-1}$.[4] If a single value of $dF_{\uparrow}/dT$ has to be used in a model for all latitude zones [as in (5)], our best guess would be this value of 1.83. However, we think it would be unrealistic to assume that it would remain constant for the range of possible climatic changes we might study.

As a range of possible values for an effective global value of B to be used in (5), we note that Ramanathan *et al.* (1976), using a radiative-convective model with fixed relative humidity, showed that

$$\frac{dF_{\uparrow}}{dT} = \begin{cases} 2.25 \text{ W m}^{-2} \,(^{\circ}\text{C})^{-1}, & \text{assuming fixed cloud-top altitude} \\ 1.37 \text{ W m}^{-2} \,(^{\circ}\text{C})^{-1}, & \text{assuming fixed cloud-top temperature.} \end{cases}$$

We have taken these values as well as the intermediate values of 1.57 (Cess, 1976) and 1.83 (from our all-season all-latitude fit) and plotted in each case the curve of ϕ_{ice} vs Q (Fig. 6). (In each case the observed global average surface temperature is matched by adjusting A.) The shape of the curve is seen to be highly dependent on the value of B. A change of B from 1.57 to 1.83 can increase the "global stability" from 1.5% to 3.2% for $f = 0.009$ and from 10.2% to 14.6% for $f = 0.004$.

A latitudinally dependent value of B, as indicated by the monthly data for 30–90°N (Table 3), might lead to increased stability for the tropics relative to other latitudes. On the other hand, if the very large values of B for the Antarctic ($B \approx 3$, see Table 3) are characteristic of glaciated continents, then we might expect the effective *global* value of B to increase if the ice line moved toward the equator, which could also increase global stability.

We note that the planetary emissivity was probably different for the prebiotic earth, because of the large amounts of IR absorbers in that atmosphere. These differences have been suggested as an explanation for the fact that the earth was never completely ice covered even 4×10^9 years ago, when the sun was probably 30% less luminous than at present (Sagan and Mullen, 1972; Henderson-Sellers and Meadows, 1977; Owen *et al.*, 1979). If the linear IR formula (5) were to be used for such a prebiotic atmosphere, both A and B would likely differ greatly from present values. If so, the curves in Fig. 6 would not be relevant for the prebiotic earth because, for all of them, A has been tuned such that the present solar constant gives the present ice line.

4. Summary

Budyko's (1969) linear heat-transport parameterization is seen to work rather well for predicting annual zonal values of heat-flux divergence. We have reevaluated it using a direct and independent test; namely, its ability to generate the monthly signal of observed net transport. We find that it works rather well for high latitudes (above 50°N) but has no predictive ability for the seasonal signal in low latitudes.

[4] These estimates are based on the lack of fit of points to the line (Mendenhall and Scheaffer, 1973, p. 389). We also estimated the uncertainties using a different procedure. We applied random perturbations to all 216 points, the perturbations being normally distributed and such that $\sigma(T) = 1$ K (our guess of zonal temperature uncertainty) and $\sigma(F_{\uparrow}) = 12$ W m^{-2}. The linear regression was done on the perturbed values. 100 sets of perturbations were done to obtain 100 A's and B's; their distribution turned out to be such that $\sigma(A) = 1$ and $\sigma(B) = 0.04$, i.e., about the same as the standard estimates. These estimates for $\sigma(A)$ and $\sigma(B)$ are quite sensitive to a factor of 2 change in our specified $\sigma(F_{\uparrow})$ but insensitive to such a change in $\sigma(T)$.

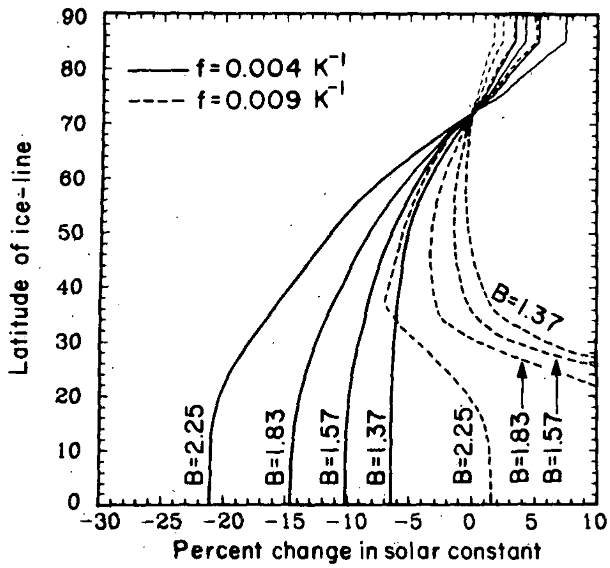


FIG. 6. Effect of changes in the infrared parameter B. The latitude of the ice line is plotted as a function of percent change in solar constant from its present value (as in Fig. 6a), for B = 1.37, 1.57, 1.83 and 2.25 W m^{-2} (°C)$^{-1}$. C = 3.74 W m^{-2} K^{-1} throughout. The value of A was chosen in each case such that the model would reproduce the present observed annual mean Northern Hemisphere surface temperature, as follows:

$$F_{\uparrow} = \begin{cases} 217.5 + 1.37T \\ 214.5 + 1.57T \\ 210.6 + 1.83T \\ 204.3 + 2.25T. \end{cases}$$

Dashed lines: f = 0.009 K^{-1}; solid lines: f = 0.004 K^{-1}.

A proposed modification to the Budyko transport parameterization (Lindzen and Farrell, 1977) was found to generate less realistic total net transports than the unaltered model, although it simulated the seasonal deviations from the annual average better than the unaltered model in one of the nine Northern Hemisphere zones.

"Global stability" of an energy-balance climate model can be increased by weakening the albedo-temperature feedback or strengthening the IR temperature feedback as well as reducing the efficiency of meridional heat transport. If we incorporate either a reduced albedo-temperature positive feedback, as suggested by Lian and Cess (1977), or an increased IR temperature negative feedback suggested by the all-month, all-latitude data used here, then the global stability of the model increases to 10 or 3%, respectively. If we incorporate both modifications, but *without* changing the transport parameterization, the global stability can approach 15%. But, as we have pointed out, the values of the IR parameterization coefficient B and of the albedo-temperature feedback coefficient f are both uncertain. Data we have analyzed can produce values of B anywhere in the range 1.6–2.3 W m^{-2} (°C)$^{-1}$ for the globe, and 1.7–3.3 for individual zones. We have not ourselves estimated values for f, but we note that other workers obtain values[5] as different as 0.009 and 0.004 K^{-1}. Because of this uncertainty in the coefficients, the functional forms of the parameterizations themselves (e.g., the linear dependence on surface temperature) are as yet speculative; and, in particular, we have shown that both Budyko's and Sellers' simple albedo formulations are unrealistic under some circumstances. We simply want to reiterate that a value of global stability much larger than 2% is not implausible, even without making modifications to the transport term. Thus, as Gal-Chen and Schneider (1976) concluded earlier, the incorporation of a more detailed transport parameterization into the model will not necessarily make the sensitivity of this model more realistic until the albedo and infrared feedbacks are known with similar accuracy. And since we have given reason also to question the form of the transport parameterization, the total uncertainty of the graph of ice line versus solar constant could be even greater than that associated with the radiation parameterizations alone. For example, the possibility that additional stability may also result from a future improved parameterization of tropical heat transport cannot be ruled out.

We think that major research emphasis should be put on improving the parameterizations of energy-balance models, in order to reduce the uncertainties discussed in this paper, while still retaining the computational efficiency of the models. Parameterizations for highly parameterized models can be tested in at least three ways: 1) against real data; 2) against simulation data from more highly resolved models; and 3) by simulation performance of a model with different parameterizations to search for those parameterizations which optimize the overall skill of a model against observations.

Despite the problems we have outlined, we do believe further work with energy-balance models can still be justified, particularly where their computational efficiency can be exploited. For example, they are the only tools available for long-term integrations or for time-scale matching experimentation in which subcomponents such as atmosphere, upper and lower oceanic layers, continental glaciers or the biosphere are simultaneously included, each with order-of-magnitude different characteristic time scales. In particular, seasonal forcing, volcanic forcing and Milankovitch experiments are appropriate problems for energy-balance

[5] These estimates were based on available observational data for the present climate. However, for large climatic changes it is possible that f could be outside this range, and possibly even negative (if cloud amounts would increase as temperature increased).

models, especially since these known forcings have been associated with measured responses. The performance of various models, especially to seasonal forcing, will be a major factor in verifying their sensitivity performance (e.g., Thompson and Schneider, 1979).

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