Parameterization of Outgoing Infrared Radiation Derived from Detailed Radiative Calculations

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ABSTRACT

State-of-the-art radiative transfer models can calculate outgoing infrared (IR) irradiance at the top of the atmosphere (F) to an accuracy suitable for climate modeling given the proper atmospheric profiles of temperature and absorbing gases and aerosols. However, such sophisticated methods are computationally time consuming and ill-suited for simple vertically-averaged models or diagnostic studies. The alternative of empirical expressions for F is plagued by observational uncertainty which forces the functional forms to be very simple. We develop a parameterization of climatological F by curve-fitting the results of a detailed radiative transfer model. The parameterization comprises clear-sky and cloudy-sky terms. Only two parameters are used to predict clear-sky outgoing IR irradiance: surface air temperature (T_s) and 0–12 km height-mean relative humidity ($\widehat{\mathrm{RH}}$). With this choice of parameters (in particular, the use of $\widehat{\mathrm{RH}}$ instead of precipitable water) the outgoing IR irradiance can be estimated without knowledge of the detailed temperature profile or average lapse rate. Comparisons between the clear-sky parameterization and detailed model show maximum errors of ~ 10 W m^{-2} with average errors of only a few watts per square meter. Single-layer "black" clouds are found to reduce the outgoing IR irradiance (relative to clear-sky values) as a function of $T_s - T_c$, T_c and $\widehat{\mathrm{RH}}$, where T_c is the cloud-top temperature. Errors in the parameterization of the cloudy-sky term are comparable to those of the clear-sky term.

1. Introduction

a. Basis for present study

It is difficult to overstate the importance of the relationship between outgoing infrared irradiance at the top of the atmosphere (F) and the Earth's surface temperature for determining global climate (Cess, 1976; Warren and Schneider, 1979). The change in surface/tropospheric temperature required to balance the outgoing infrared for an internal perturbation (e.g., clouds) or an external one (e.g., solar output) is a measure of the sensitivity of the climate system to the perturbation.

In principle it should be possible to calculate the outgoing infrared (IR) irradiance to a satisfactory degree of accuracy given the vertical distributions of temperature and optically active constituents. In practice, however, inadequacies primarily in the atmospheric specification (in particular, clouds) limit the accuracy with which F may be calculated (Warren and Thompson, 1983). Deficiencies in detailed IR models also contribute to the overall error. However, Ellingson and Gille (1978) have estimated this error to be at most 3% for a clear-sky outgoing irradiance calculation, which is comparable to the uncertainties introduced by using conventional radiosonde profiles.

Regardless of the precise level of error inherent in detailed infrared models there is a practical need for parameterizations of F suitable for inclusion in low-resolution climate models (e.g., Thompson and Schneider, 1979) or for use in diagnostic studies of GCM results (e.g., Ramanathan, 1977). In addition, a reliable parameterization of F should prove useful to climatic theory by highlighting the dominant factors controlling changes in F.

Most previous approaches to parameterizing F have been essentially empirical. [Cess (1974) developed a simple analytical formulation for F but included only radiative transfer due to water vapor.] The empirical studies have regressed the outgoing IR irradiance observed from satellites against various climatological variables. A common form of such a parameterization is

$$F = A + BT_s, \tag{1}$$

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where A and B are empirically determined constant coefficients and T_s is surface (or sometimes sea level) air temperature. Eq. (1) has been used by Oerlemans and van den Dool (1978), Thompson and Schneider (1979), North *et al.* (1981) and other researchers. Table 3 of Warren and Schneider (1979) illustrates the inadequacy of (1) as a general formula by showing that different empirical studies can give a wide range of values for B, differing from each other by as much as a factor of 2. A modification of (1) originated by Budyko (1969) and adopted by Cess (1976) is

$$F = A + BT_s + CA_c\,, \tag{2}$$

where A, B and C are constant coefficients and A_c is the fractional total cloud amount. Modifications can be extended by making C a function of T_s but Cess (1976) found this effect to be small based on annually-averaged observations. Eq. (2) can be rewritten in the general form for F used by Cess (1974):

$$F = c_1 - c_2A_c\,, \tag{3}$$

where c_1 and c_2 can be functions of T_s or other variables. The outgoing IR irradiance for clear-sky conditions is given by c_1; c_2A_c is the modification term which accounts for the effect of clouds in decreasing F. Alternatively, Eq. (3) may be rewritten as

$$F = c_1(1 - A_c) + c_2'A_c\,, \tag{4}$$

where $c_2' = c_1 - c_2$. This form seems physically more straightforward than (3) but (3) is often used because $c_2 = -\partial F/\partial A_c$; c_2 will be positive in the normal situation where the cloud top is colder than the surface. [Eq. (4) assumes that cloud height-to-width ratios are small. A better assumption would be to use an "infrared effective cloud fraction" as in Fig. 4 of Harshvardhan and Weinman (1982). Such an effective cloud fraction is actually close to the definition of "cloud cover" used by ground observers, in which cloud sides as well as bottoms are counted (WMO, 1969, p. 35).]

There are some intractable difficulties in the observation-based approach to deriving a parameterization for F. It seems that after more than a decade of empirical attempts we still do not know B in Eq. (2) to better than 20% (Warren and Schneider, 1979). Modeling studies have been necessary to clarify and establish values for C (Ramanathan, 1977; Cess and Ramanathan, 1978). A major reason for this difficulty is our imperfect knowledge of T_s, A_c and F. Observational errors in these quantities contribute to errors in regression coefficients. This observational "noise" makes it hard to improve on very simple functional forms like (2) due to a lack of statistical significance for subtle modifications to the functions which one fits to the observations (e.g., making C a function of cloud-top temperature). A related problem arises when two physically distinguishable but linearly correlated factors influence F. It is then quite difficult to separate them and establish their relative magnitudes by regression analysis.

Considering these problems with the observational approach, we believe a parameterization for F based on the results of a detailed infrared model is justifiable. The relative accuracy of state-of-the-art models is sufficient to provide a useful alternative to observations of F. The advantage to this approach is that the input atmospheric conditions and output F are known perfectly. Furthermore, in a model it is relatively easy to separate contributions due to individual processes, even if the processes are correlated in nature.

Disadvantages to a model-derived parameterization result from processes excluded from the model or imperfectly included. In addition, an observational approach might implicitly account for unsuspected feedback effects in the climate system that might otherwise be neglected in a climate model. For these reasons the modeling approach can only be considered as a complementary alternative to the empirical approach, not a definitive substitute.

Our goal is to develop a parameterization of outgoing infrared irradiance at the top of the atmosphere which will be useful for climate theory and for climate models incorporating a vertically-integrated atmosphere. [We do not require it to be of such simple form that it can be used by analytically solvable climate models, as (1) can.] The parameterization will be derived using the results of a detailed model but will have very few degrees of freedom (i.e., constant coefficients and input parameters) relative to the detailed model. After describing our model and data sources we begin by performing some calculations with the detailed model to demonstrate the feasibility of parameterizing the clear-sky component [c_1 in (3)]. We then derive the c_1 parameterization and compare its results to those of the detailed model. Finally, we consider the effects of clouds on F and use the detailed model to derive a cloud modification function c_2.

b. Model description

Radiative transfer calculations of outgoing IR at the top of the atmosphere (TOA) are performed using the atmospheric radiation model of Wiscombe (1975). This model (ATRAD) employs the exponential-sum-fitting-of-transmission-functions method (reviewed by Wiscombe and Evans, 1977) to handle line absorption. This method allows spectral intervals containing many absorption lines to be treated reasonably correctly even when there is scattering or surface reflection (the latter is important in the infrared when surface emissivity $\epsilon < 1$). However, since the model was employed in the present context only for clear skies or blackbody clouds, only the no-scattering limit for the Planck function source doubling (Wiscombe,

1976) was needed. In this circumscribed usage ATRAD is like other sophisticated longwave models (e.g., Ellingson and Gille, 1978) save in one respect only: by properly handling the diffusion or optically-thick limit (see Wiscombe, 1976), the model need not take very small vertical steps. In fact, for all of our calculations we used only the 31 levels in the well-known McClatchey *et al.* (1972) model atmospheres, consisting of 1 km steps up to 25 km and 5 km steps above that up to 50 km. We employ 136 spectral intervals spanning $3 \leqslant \lambda \leqslant 500$ μm, since the model was already set up for such a spectral resolution. A coarser resolution would have been quite satisfactory but would have entailed making a new exponential fit table.

The transmission functions in ATRAD are principally those of LOWTRAN5 (Kneizys *et al.*, 1980). LOWTRAN5, however, includes the water-vapor continuum of Roberts *et al.* (1976) only for $7 \leqslant \lambda \leqslant 15$ μm, but ATRAD includes the continuum absorption over a wider spectrum, $7 \leqslant \lambda \leqslant 30$ μm, i.e., the entire spectral range measured by Burch (Burch and Gryvnak, 1979), using the empirical fit given by the solid line in Fig. 2 of Roberts *et al.* (1976). This continuum absorption is proportional to the square of the water-vapor concentration. It is due to absorption by dimers of H_2O or to self-broadened wings of strong lines in the H_2O rotation band at $\lambda > 30$ μm, and is included in addition to other water-vapor absorption lines whose centers are in the 7–30 μm region. The water-vapor continuum absorption assumed in ATRAD is essentially similar to what will be included in the new LOWTRAN6, which will be based on measurements of the continuum (and calculations for the far-wing line shapes) over most of the spectrum from the visible to the microwave, both pressure-broadened and self-broadened (Kneizys, personal communication to Wiscombe, 1981). [Warren and Thompson (1983) give a more detailed description of the treatment of the H_2O continuum.]

c. *Data sources and assumptions*

Vertical profiles of temperature (T), ozone and relative humidity (RH) for the clear-sky feasibility tests and derivation of c_2 are given by the five model atmospheres of McClatchey *et al.* (1972). The McClatchey RH profiles are shown in Fig. 1. Ramanathan (1977, his Fig. 2) shows the temperature profiles.

For the actual derivation and testing of the clear-sky parameterization c_1 we need more data than is afforded by the five model atmospheres. We therefore compile a set of zonally averaged T, RH and O_3 profiles for January, April, July and October for all 5° latitude zones 75°N–90°S (as well as some fictitious profiles more extreme than any of these). These profiles are constructed as follows, and are the same as those used by Warren and Thompson (1983).

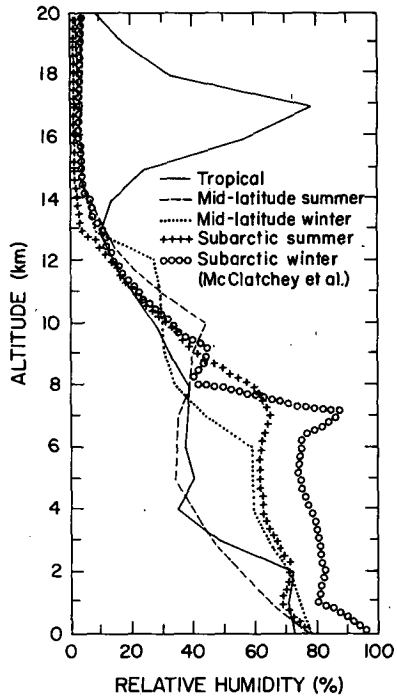

FIG. 1. Relative humidity profiles of the model atmospheres of McClatchey *et al.* (1972). [Temperature profiles are shown in Fig. 2 of Ramanathan (1977).]

Surface air temperatures are given in Table 1a of Warren and Schneider (1979). Temperatures for 1–22 km are taken from Taljaard *et al.* (1969) for 0–85°S and from Oort and Rasmusson (1971) for 5–75°N. Values for our height levels were interpolated from the values given at pressure levels by integrating the hydrostatic equation with knowledge of the zonal mean surface elevation. The 50 mb temperatures for the Northern Hemisphere (NH) were assumed to apply with seasonal reversal to the Southern Hemisphere (SH) (where the data of Taljaard *et al.* do not extend above 100 mb) except for 80–90°S where the temperatures were obtained by interpolation between 75°S and the South Pole 50 mb temperatures in Fig. 6 of Crutcher (1969). Temperatures and relative humidities above 22 km were taken from the model atmospheres of McClatchey *et al.* (1972), interpolated in latitude ϕ for $15° < |\phi| < 70°$ with the assumption that "subarctic" profiles apply to $|\phi| \geqslant 70°$, "mid-latitude" to $|\phi| = 45°$, and "tropical" to $|\phi| \leqslant 15°$. Ozone profiles were taken from a similar interpolation of the McClatchey model atmospheres.

Zonal average relative humidities (RH) below 22 km were either taken or derived from Oort and Rasmusson (1971) for 0–7 km, 10°S–75°N; from London (1957) for 7–22 km, 0–75°N; and from Sasamori *et al.* (1972) for 0–22 km, 10–90°S and 7–22 km, 0–10°S.

In addition we make the following assumptions:

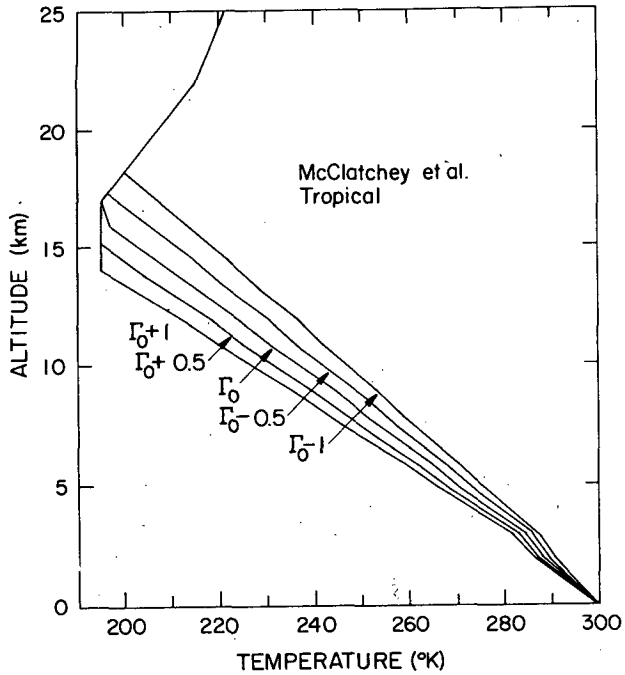

FIG. 2. Temperature lapse rate perturbations applied to the McClatchey *et al.* (1972) Tropical profile. Γ_0 refers to the standard profile. The standard profile lapse rate is perturbed by ± 0.5 K km^{-1} and ± 1.0 K km^{-1} up to tropopause height.

uniform constant CO_2 = 320 ppmv, ground surface temperature equals surface air temperature, and a constant surface emissivity of 0.95. (The fact that surface emissivity is less than unity causes the surface to reflect some of the IR radiation incident on the surface from the atmosphere; this is treated properly in the model.) These assumptions are discussed in more detail in Section 2c.

2. Clear-sky feasibility tests

Obviously, many factors influencing F cannot be included in a simple parameterization, in particular details of the vertical temperature and humidity profiles. We first show that knowledge of only surface temperature and a measure of the tropospheric mean relative humidity may be sufficient to allow a reasonably accurate estimate of clear-sky outgoing IR irradiance.

a. Effects of the temperature profile

The effect of the temperature lapse rate Γ on outgoing IR irradiance is tested by perturbing Γ in the McClatchey profiles. An example of the Γ perturbation process is shown in Fig. 2 for the Tropical atmosphere $T(z)$. For each unperturbed profile (Γ_0) there are four perturbed profiles with $\Gamma = \Gamma_0 \pm 0.5$ and $\Gamma_0 \pm 1.0$ K km^{-1} at all levels up to the tropopause. Stratospheric temperatures are held fixed. The perturbed tropopause level is defined as the point where the perturbed profile reaches the unperturbed tropopause temperature (for $\Delta\Gamma > 0$) or at the point where the perturbed profile intersects the unperturbed lower stratospheric profile (for $\Delta\Gamma < 0$). The other model atmospheres are perturbed in a similar manner. For each perturbed $T(z)$ profile we consider two assumptions for humidity: 1) fixed mixing ratio (w) or 2) fixed relative humidity. It should be emphasized that we do not use these hypothetical perturbed temperature profiles to derive the parameterization for c_1. In the derivation we use climatological profiles (see Section 3a).

Fig. 3 shows the results of the detailed model calculations of the Mid-Latitude Summer and Tropical atmospheres. (The other three model atmospheres show smaller ranges in outgoing IR for Γ perturbations.) Note that the assumption of fixed RH results in a much smaller $\Delta IR/\Delta\Gamma$ than does the assumption of constant mixing ratio. For the Tropical atmosphere the range of clear-sky IR from $\Gamma_0 - 1$ to $\Gamma_0 + 1$ is

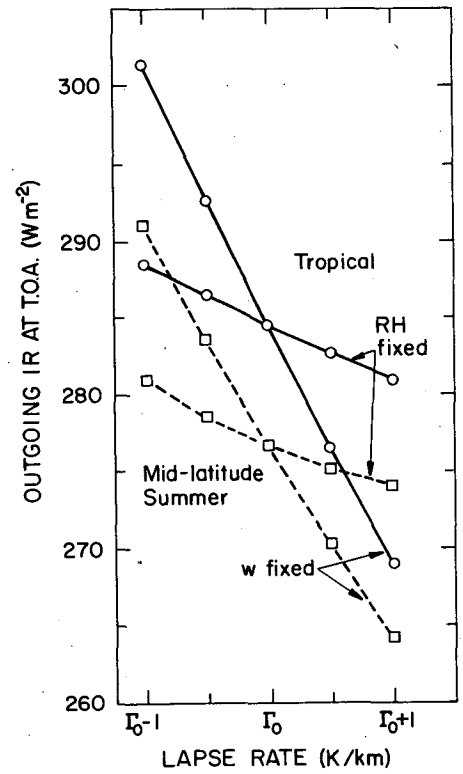

FIG. 3. Outgoing clear-sky infrared irradiance as a function of lapse rate for perturbations of two of the McClatchey temperature profiles, each using two humidity assumptions. Γ_0 refers to the standard lapse rate of the profile. Dots refer to calculated points; straight lines are drawn connecting them for ease of interpretation. Note that the assumption of fixed relative humidity (RH) results in a much smaller variation of outgoing IR than does the assumption of fixed mixing ratio (w). [The other three McClatchey model atmospheres show smaller ranges in outgoing IR than do these two.]

32.5 W m^{-2} for fixed w but only 7.5 W m^{-2} for fixed RH.

The explanation for this behavior is straightforward. For fixed w there is little change in atmospheric opacity with a change in Γ; thus increasing Γ cools the atmosphere and decreases the outgoing IR considerably. However, for fixed RH the atmospheric opacity decreases as Γ is increased because the colder atmosphere contains less water vapor. Decreasing the atmospheric opacity means that more IR comes from the relatively warm surface and less from the relatively cool atmosphere. The competing effects of temperature change and opacity change constitute a "Γ-opacity compensation" that makes $\Delta IR/\Delta\Gamma$ fairly small for fixed RH. Since fixed RH(z) is usually considered to be a better climate modeling assumption than fixed $w(z)$ (e.g., Manabe and Wetherald, 1967), it should be possible to neglect tropospheric lapse rate variations of up to 1 K km^{-1} in a parameterization without incurring errors in outgoing clear-sky IR of more than 4 W m^{-2}.

It would be convenient to separate the atmospheric and surface contributions to c_1 and thus allow a variable surface emissivity in the parameterization. However, the Γ-opacity compensation effect which works well for total outgoing clear-sky IR does not apply to the separate contributions. Fig. 4 shows the percentage change in clear-sky IR versus the perturbation of Γ for the Mid-Latitude Winter profile. (This profile is shown because it has the largest surface contribution to outgoing IR of any of the five profiles.) Note that while the individual surface and atmospheric contributions vary by ~ ±4%, their sum varies by only ~ ±1%. Thus a separate parameterization of the surface and atmospheric components, while allowing greater versatility, would likely lead to large errors in each component unless lapse rate changes were somehow accounted for. For this reason we choose to parameterize *total* clear-sky IR and therefore take advantage of the Γ-opacity compensation.

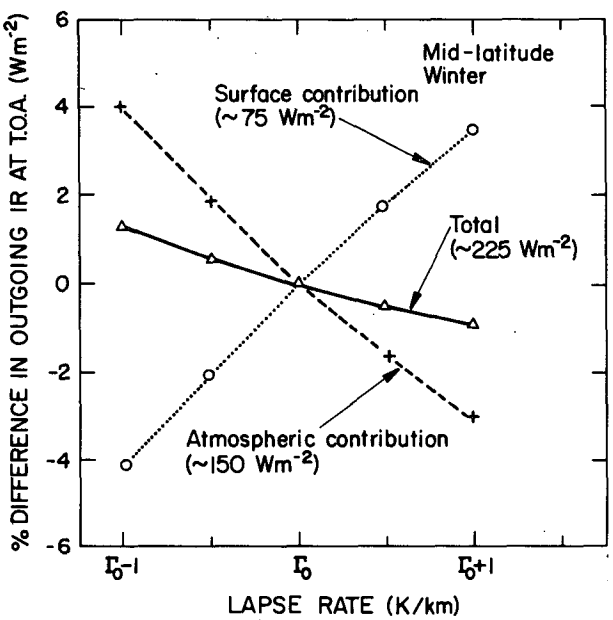

FIG. 4. Percentage differences in the surface contribution, atmospheric contribution and total clear sky outgoing IR irradiance at the top of the atmosphere as a function of lapse rate for the Mid-Latitude Winter profile. [Percent difference is defined as $[IR(\Gamma) - IR(\Gamma_0)] \cdot 100/IR(\Gamma_0)$, where Γ_0 is the standard lapse rate of the profile.] Dots refer to calculated points.

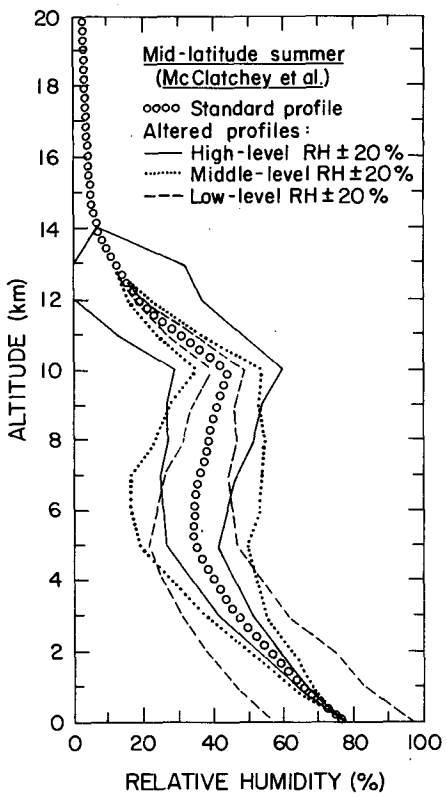

FIG. 5. Non-uniform perturbations of relative humidity applied to the McClatchey *et al.* (1972) Mid-Latitude Summer relative humidity profile. Small circles show the unperturbed standard profile.

b. *Effects of the humidity profile*

We now consider the sensitivity of the outgoing clear-sky IR to the vertical structure of the water vapor profile. Again we perform a set of perturbation experiments, but this time hold $T(z)$ fixed and perturb tropospheric RH(z). An example of the RH perturbations we employ is shown in Fig. 5 for Mid-Latitude Summer. Several perturbation profiles are used: uniform with height of ±5%, ±10%, ±20% (not shown in Fig. 5); and non-uniform with height comprising "low-level" with ±20% at the surface decreasing to zero at the tropopause, "high-level" (the reverse of "low-level"), and "mid-level" with ±20% in the mid-

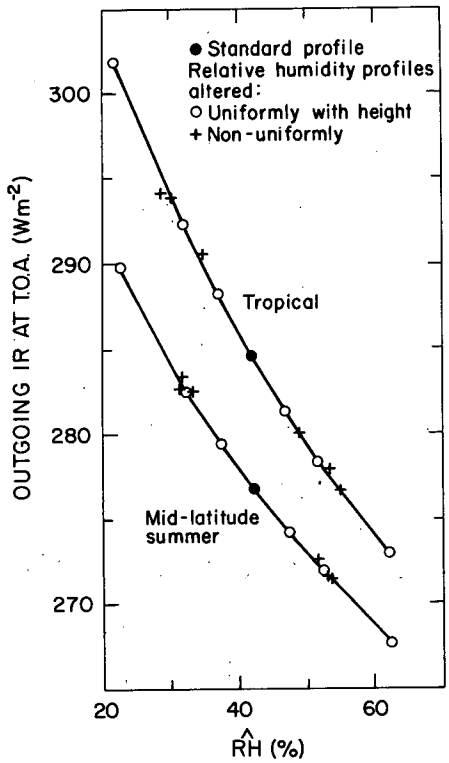

FIG. 6. Outgoing clear sky IR irradiance as a function of the 0–12 km height-mean relative humidity ($\widehat{RH}$). Smooth lines are drawn to connect the open circles referring to RH perturbations which are uniform with height. Solid dots refer to unperturbed RH profiles; plus signs to non-uniform perturbations.

dle troposphere decreasing to zero at the tropopause and surface.

Clear-sky outgoing IR is computed for each temperature profile with its standard RH(z) and its 12 perturbed RH profiles. We find a substantial range of outgoing IR for any given $T(z)$ profile depending on RH(z) (e.g., a range of 29 W m^{-2} for uniform ±20% RH perturbations to the Tropical profile). Since we obviously cannot ignore IR variations this large, we would like to find a single measure or parameter of the water vapor profile that allows us to estimate the outgoing IR. Since the atmospheric IR opacity tends to increase roughly logarithmically with H_2O concentration (e.g., Cess, 1974, Fig. 1), changes in water vapor concentration in the upper troposphere have a larger effect on outgoing IR than do equivalent changes in the lower troposphere. Thus for estimating outgoing IR, water vapor variations in a given pressure interval must be weighted more heavily in the upper troposphere.

We choose RH, rather than absolute humidity, as our basic moisture variable. We do this because RH varies over a much smaller range in reality than does absolute humidity. Therefore, in a parameterization using RH one would be much more justified in assuming a constant RH than a constant absolute humidity. In any case, RH is easily related to other moisture variables. We have tried three types of weighted vertically-averaged RH: (A) a height-mean $\widehat{RH}$, (B) a density-weighted RH, and (C) the ratio of precipitable water to saturation precipitable water. The latter two averages have an advantage of being insensitive to the upper height limit of the integration, while the first average preferentially weights upper tropospheric absolute humidity variations. We find that $\widehat{RH}$ defined as the height-mean relative humidity from 0 to 12 km allows us to plot outgoing clear-sky IR vs $\widehat{RH}$ for any of the McClatchey temperature profiles with very little scatter about a smooth curve, i.e.,

$$\widehat{RH} \equiv (Z_t - Z_b)^{-1} \int_{Z_b}^{Z_t} RH(z)dz, \qquad (5)$$

where Z_b = 0 km and Z_t = 12 km. The upper limit is an approximate mean tropopause height; this choice of Z_t also minimizes the deviations of the points from the curve in Fig. 6.

Fig. 6 shows the outgoing clear-sky IR as a function of $\widehat{RH}$ for the Tropical and Mid-Latitude Summer $T(z)$. These profiles have the largest range of IR for RH perturbations. Open circles connected by smooth lines represent the outgoing IR for uniform RH perturbations. Note that the non-uniformly perturbed cases all fall very close to the lines. This indicates that $\widehat{RH}$ may be a good measure of the effect of RH(z) on clear-sky IR, even for non-uniform (yet still fairly

TABLE 1. Justification for use of $\widehat{RH}$ as the definition of vertically averaged relative humidity. Given are the deviations in outgoing IR (W m^{-2}) due to non-uniform perturbations in RH(z) from Fig. 5, relative to uniformly perturbed McClatchey Tropical profiles having the same change in vertical mean humidity. (Note: These are *not* deviations of F relative to the standard profile, which are shown in Fig. 6 to be much larger, up to 18 W m^{-2}.) Three definitions of vertical mean relative humidity are compared:

A. Height-mean relative humidity ($\widehat{RH}$).
B. Density-weighted height-mean (i.e., pressure-mean) relative humidity.
C. Precipitable water divided by precipitable water in a saturated atmosphere.

The values in column A correspond to the deviations of the plus signs from the solid line in Fig. 6.

Non-uniform RH perturbation*		Definition of vertical mean		
		A	B	C
Low	−20%	−1.0	−3.1	−5.3
Middle	−20%	−0.1	+2.2	+5.5
High	−20%	+0.6	+2.8	+4.1
High	+20%	0.0	−1.4	−2.8
Middle	+20%	+0.6	−0.6	−3.2
Low	+20%	+0.1	+1.3	+1.6

* Listed in order of decreasing F.

smooth) changes in climatological tropospheric humidity profiles.

Table 1 justifies the use of $\widehat{RH}$ rather than one of the other vertical averaging definitions. For our nonuniform RH(z) perturbations, using $\widehat{RH}$ as a predictor of outgoing IR results in no more than 0.6 W m^{-2} error. Using the other vertical averaging definitions results in maximum errors of −3.1 and +5.5 W m^{-2} for definitions B and C respectively.

A climatology of $\widehat{RH}$ has been prepared from the zonally-averaged data of Oort and Rasmusson (1971), London (1957) and Sasamori *et al.* (1972, Fig. 2.3) described in Section 1c. Shown in Fig. 7 is $\widehat{RH}$ by latitude and month. Of particular interest is the belt of high $\widehat{RH}$ near the equator surrounded by low $\widehat{RH}$ in subtropical regions. The tropical high of $\widehat{RH}$ migrates seasonally in a manner similar to that of the ITCZ (with which it is probably associated) and has been shown to contribute significantly to the dip in observed tropical outgoing IR (Warren and Thompson, 1983). Table 2 gives the zonally-averaged values of $\widehat{RH}$ for January, July and the annual mean.

The values of RH and $\widehat{RH}$ used in this study are likely to be somewhat different from the correct climatological values for several reasons. For the Southern Hemisphere, monthly mean temperatures and dew point temperatures were used by Sasamori *et al.* (1972) to compute RH. This procedure typically underestimates the monthly mean RH by roughly 0.05 [based on an analysis of radiosonde observations of the sort described in the Appendix of Warren and Thompson (1983)]. For the Northern Hemisphere, monthly mean specific humidities and temperatures were used as given by Oort and Rasmusson (1971). In contrast to the Southern Hemisphere this procedure tends to overestimate monthly mean RH. Indeed, a bias toward higher $\widehat{RH}$ in the Northern Hemisphere is evident for the mid-latitudes and subtropics (see Table 2). In addition, the RH values are means over all cloud conditions. Preferably, one should use separate RH or $\widehat{RH}$ values for clear and cloudy areas when computing F. There is clearly a need for a better climatology of RH as a function of season, latitude, height and cloud amount. However, the errors in Fig. 7 and Table 2 have no effect on the validity of the parameterization we derive since it is fitted to model calculations rather than to observations. The parameterization will still be valid for use with better RH climatologies when these become available.

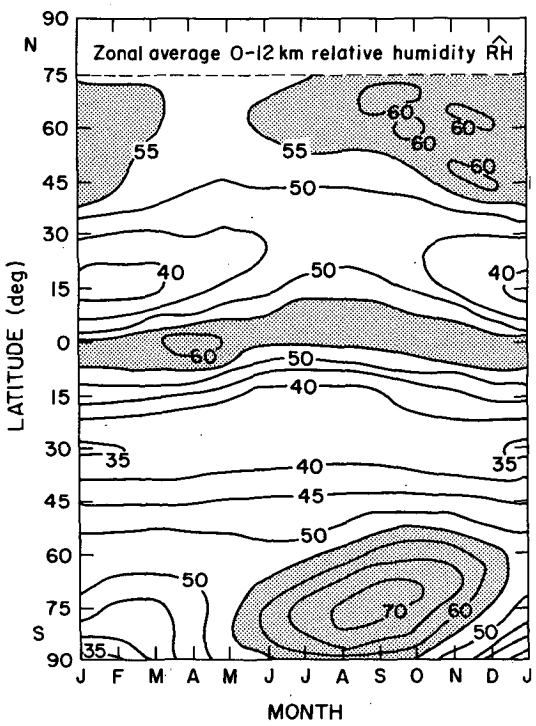

FIG. 7. Distribution of 0–12 km height-mean relative humidity $\widehat{RH}$ by latitude and month, derived from data sources given in the text. Some of these values are listed in Table 2.

TABLE 2. Fractional value of $\widehat{RH}$ derived from zonally-averaged data using Eq. (5). (The January and July values are also plotted in Fig. 7.)

Latitude	January	July	Year
75°N	0.55	0.53	0.54
70	0.57	0.55	0.57
65	0.60	0.56	0.58
60	0.60	0.55	0.57
55	0.59	0.56	0.56
50	0.59	0.54	0.55
45	0.58	0.51	0.54
40	0.56	0.48	0.51
35	0.50	0.46	0.48
30	0.46	0.45	0.46
25	0.42	0.46	0.44
20	0.40	0.49	0.44
15	0.38	0.52	0.45
10	0.41	0.56	0.50
5	0.46	0.58	0.54
0	0.57	0.55	0.58
5	0.56	0.50	0.55
10	0.51	0.42	0.48
15	0.46	0.36	0.42
20	0.41	0.36	0.39
25	0.36	0.35	0.36
30	0.35	0.37	0.36
35	0.37	0.40	0.38
40	0.41	0.44	0.42
45	0.45	0.46	0.46
50	0.49	0.48	0.49
55	0.51	0.50	0.52
60	0.53	0.53	0.56
65	0.53	0.57	0.58
70	0.51	0.63	0.59
75	0.47	0.67	0.58
80	0.45	0.68	0.57
85	0.40	0.63	0.53
90°S	0.33	0.56	0.48

c. Neglected parameters

In a parameterization of F for climate modeling studies the absolute value of F is not as important as the sensitivity of F to a parameter change. There is sufficient uncertainty in climatic observations to allow F to be tuned, within limits, to some desired value (e.g., by changing cloud amount). If a given parameter can be assumed constant or only slightly varying, or if F varies only weakly with a large variation in that parameter, then the parameter may justifiably be neglected (implicitly held fixed) in our parameterization. The parameters discussed below meet these requirements.

1) SURFACE ELEVATION

Since our parameterization is intended for climate modeling applications on large spatial scales the inclusion of surface elevation (or pressure) as a parameter is not necessary. To show this we perform calculations with the detailed model for the two largest high elevation areas on earth: the plateaus of Antarctica and Tibet. We use the climatological profiles of $T(z)$ and RH(z) for 40°N and 90°S for January and July; i.e., we fix T_s, not sea level temperature, as we alter the surface elevation. Surface pressures of 700 and 1013 mb are tested for both plateaus. For Antarctica the difference in clear-sky outgoing IR between p_s = 700 mb and p_s = 1013 mb is only ~0.5 W m^{-2}. For the Tibetan plateau the differences are ~8 W m^{-2}. This difference is fairly substantial, but would be smaller if we would use Tibetan surface temperatures rather than the 40° zonal mean surface temperature. Furthermore, in a zonally-averaged application the surface elevation bias introduced by the Himalayas and Tibetan plateau would not be significant.

2) SURFACE EMISSIVITY

Our assumption of 0.95 for surface emissivity is intermediate between the extremes of 0.90 for sand desert and 0.99 for snow measured by Griggs (1968) in the 8–14 μm region. The emissivity of a water surface in this region is about 0.95, according to calculations of Miranova (1973, her Table 4.44). Outside the 8–14 μm window the possible variation of surface emissivity has little effect on the outgoing IR radiation from the surface, which is the sum of the emitted and reflected radiation. We do not allow for variable surface emissivity in the parameterization, because this would require separating the atmospheric and surface components of F. As discussed above, we do not separate them because it is much more difficult to parameterize the separate components than to parameterize their sum.

Test calculations performed with the detailed model show a variation in clear-sky outgoing IR of 1–3 W m^{-2} for a change of 0.05 in surface emissivity, depending on atmospheric opacity. In a climate model, such a change in F would lead to a surface temperature change on the order of 1 K. However, a change in surface emissivity of 0.05 implies a substantial climate change (e.g., going from a vegetated surface to a snow field); thus the "surface emissivity feedback" would be relatively weak.

3) DISCONTINUITY OF TEMPERATURE AT THE SURFACE

For diurnal means and large space scales the discontinuity between the surface and surface air (screen height) temperatures is normally a few kelvins. Thus a 1 K change in this parameter could be considered quite large. Test calculations show only a 0.7–1.5 W m^{-2} change in outgoing clear-sky IR for a 1 K change in temperature discontinuity at the surface. For this reason we neglect the discontinuity and assume equal surface and surface air temperatures. Our assumption, however, probably means that the parameterization will not accurately follow the course of the diurnal cycle of clear-sky IR over land, especially in regions of low $\widehat{\mathrm{RH}}$.

4) CARBON DIOXIDE

Considering the prominence of the question of possible climatic effects from an anthropogenic atmospheric CO_2 increase it would seem reasonable to allow for variation of the CO_2 concentration in our parameterization. Ramanathan *et al.* (1979) show that the important quantity for determining the impact of CO_2 variations on surface temperature is the net change in surface-tropospheric radiative heating, i.e., the net change in IR at the tropopause. (This is true if the troposphere is well mixed and the surface is strongly coupled to it, as in radiative-convective models.) However, CO_2 variations influence stratospheric opacity to the extent that the change in outgoing IR at the top of the atmosphere is roughly a factor of 2 smaller than the net change in IR at the tropopause. Since our parameterization does not separate the stratospheric and tropospheric contributions to outgoing IR, we feel that including CO_2 concentration as a parameter would not be useful. This is not to say that simple vertically-averaged models cannot be used in studies of the CO_2 climate problem, just that such models should impose a change in effective radiative forcing derived from radiative-convective or general circulation models [as was done, for example, by Schneider and Thompson (1981)], instead of calculating it in the parameterization for F.

3. Clear-sky parameterization

a. Development

The feasibility tests above indicate the possibility of developing a parameterization of clear-sky out-

going IR that uses only surface air temperature and the humidity parameter $\widehat{\mathrm{RH}}$. To derive the parameterization we use the zonally-averaged temperature and humidity data (Section 1c) for January and July. For each temperature profile the RH profile is perturbed uniformly with height, as in the feasibility tests. This process gives 11 RH(z) profiles for each $T(z)$ with $\widehat{\mathrm{RH}}$ = 0, 10, 20, . . . , 100%. (RH at any height is not allowed to exceed 100% nor to fall below the value corresponding to the assumed stratospheric mixing ratio of 2.5×10^{-6}). For climate modeling purposes we want the parameterization to be valid somewhat beyond the present climatological range of T_s. Therefore, we create some hypothetical temperature profiles to extend the temperature range. The upper limit of T_s is extended in steps of 5 to 330 K by uniformly increasing the tropospheric temperature of the warmest tropical profile. Similarly, we decrease T of the coldest polar profile to extend the lower limit of T_s in steps of 20 K to 155 K. (Admittedly the radiation model may become somewhat inaccurate at these extreme temperatures, since the transmission functions of LOWTRAN are fixed for 273 K so that dependence of absorption line strength on temperature is ignored.)

For each $T(z)$, both observed and hypothetical, we use the detailed model to compute 11 values of outgoing clear-sky IR corresponding to the 11 $\widehat{\mathrm{RH}}$ values. Fig. 8 shows the resulting values of IR for two of these, $\widehat{\mathrm{RH}}$ = 20% and 100%, plotted against T_s. As the feasibility tests indicated, the outgoing IR tends to be a function of T_s alone only for fixed $\widehat{\mathrm{RH}}$.

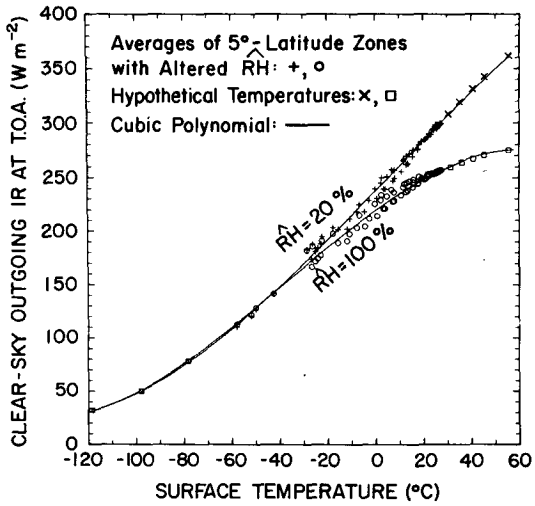

FIG. 8. Outgoing clear sky IR irradiance plotted against surface temperature for assumptions of $\widehat{\mathrm{RH}}$ = 20% (+, ×) and $\widehat{\mathrm{RH}}$ = 100% (O, □). Plotted points (O, +) are calculated using observed zonally-averaged temperature profiles for January and July. Relative humidity profiles are perturbed uniformly with height to give the required $\widehat{\mathrm{RH}}$. Hypothetical temperature profiles (×, □) are added to extend the range of outgoing IR. The smooth curves are least-squares fitted cubic polynomials in T_s.

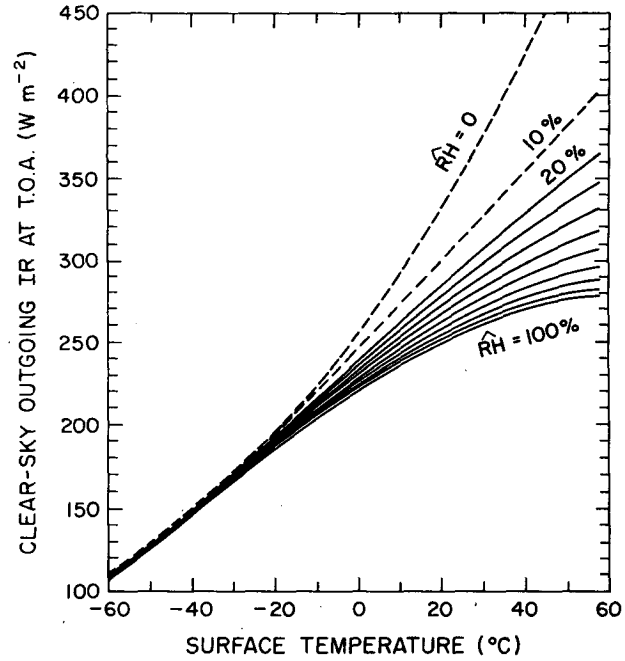

FIG. 9. Family of curves depicting the parameterization of clear-sky outgoing IR irradiance as a function of T_s and $\widehat{\mathrm{RH}}$. Dashed lines for $\widehat{\mathrm{RH}}$ = 0 and 10% are not used in deriving the parameterization function but are shown for the sake of completeness.

For very low T_s outgoing IR is insensitive to $\widehat{\mathrm{RH}}$ since extremely low absolute humidity implies a maximum in atmospheric transmissivity. However, at high T_s the points for $\widehat{\mathrm{RH}}$ = 20% and 100% diverge widely, justifying the inclusion of $\widehat{\mathrm{RH}}$ in the parameterization.

The curves in Fig. 8 are cubic polynomials in T_s fitted by a least-squares technique to the two $\widehat{\mathrm{RH}}$ cases. Quadratic fits are clearly deficient and polynomials of higher order than cubic give no significant improvement. Cubic polynomials in T_s are fitted for each $\widehat{\mathrm{RH}}$ from 20% to 100% in 10% increments. The coefficients of these nine polynomials are then regressed against $\widehat{\mathrm{RH}}$ to derive a single expression for clear-sky outgoing IR: a cubic polynomial in T_s whose four coefficients are themselves quadratic polynomials in $\widehat{\mathrm{RH}}$. This parameterization is shown as a family of curves in Fig. 9; the equation and coefficients are given in Table 3. Also shown are the curves for $\widehat{\mathrm{RH}}$ = 0 and 10%. These are not used to determine the coefficients of the parameterization because they are climatologically unlikely and would require the parameterization to use higher order polynomials in $\widehat{\mathrm{RH}}$.

An interesting feature of Fig. 9 is the obvious nonlinear relation of F to both T_s and $\widehat{\mathrm{RH}}$. For very low $\widehat{\mathrm{RH}}$, outgoing IR tends to a T_s^4 type dependence, consistent with a rather transparent atmosphere. At

TABLE 3. Parameterization for clear-sky outgoing IR irradiance at the top of the atmosphere (z = 50 km).

c_1 ≡ outgoing clear sky IR (W m^{-2}).
T_s = surface air temperature (°C) ($-118°C \leq T_s \leq 57°C$).
$\widehat{RH}$ = height-mean relative humidity from 0 to 12 km (Eq. 5) ($0.2 \leq \widehat{RH} \leq 1.0$):

PARAMETERIZATION:

$$c_1 = a_0 + a_1 T_s + a_2 T_s^2 + a_3 T_s^3, \text{ where}$$

$$a_n = b_{0n} + b_{1n}\widehat{RH} + b_{2n}\widehat{RH}^2, \quad n = 0, 1, 2, 3.$$

$n = 0$	$n = 1$
$b_{00} = 2.43414 \times 10^2$	$b_{01} = 2.60065 \times 10^0$
$b_{10} = -3.47968 \times 10^1$	$b_{11} = -1.62064 \times 10^0$
$b_{20} = 1.02790 \times 10^1$	$b_{21} = 6.34856 \times 10^{-1}$
$n = 2$	$n = 3$
$b_{02} = 4.40272 \times 10^{-3}$	$b_{03} = -2.05237 \times 10^{-5}$
$b_{12} = -2.26092 \times 10^{-2}$	$b_{13} = -9.67000 \times 10^{-5}$
$b_{22} = 1.12265 \times 10^{-2}$	$b_{23} = 5.62925 \times 10^{-5}$

high $\widehat{RH}$ the effect of increasing atmospheric opacity is evident as the slope of the curves decreases with increasing T_s. However, no downturn occurs, i.e., $\partial F/\partial T_s$ remains positive; $\partial F/\partial T_s < 0$ would indicate a radiative climatic instability (a "runaway greenhouse"), for the chosen range of temperatures. For intermediate values of $\widehat{RH}$ the parameterization tends to be fairly linear in T_s, thus supporting empirical results. A consequence of the nonlinear dependence of clear-sky IR on $\widehat{RH}$ is the tendency to bias downward the computed IR when using spatial or temporal averages of $\widehat{RH}$. This is discussed by Warren and Thompson (1983).

b. Comparison with detailed model

We gauge the accuracy of our clear-sky IR parameterization by comparing its results against those of the detailed model using the observed RH(z) profiles. Shown in Fig. 10 is the clear-sky IR as a function of latitude for the four mid-season months as parameterized and as calculated using the detailed model. The differences between the two methods are encouragingly small. Maximum differences are ~10 W m^{-2} with average differences of only a few watts per square meter. (For comparison, errors of this magnitude are similar to or less than the estimated observational uncertainty of zonally-averaged outgoing IR derived from satellite measurements.) The dip in clear-sky IR, attributable to the maximum of $\widehat{RH}$ in the tropics, is modeled well by the parameterization.

There is a tendency for a systematic difference between the two methods, the parameterization being higher than the detailed model in the winter hemisphere and lower in the summer hemisphere. We attribute this to the change in stratospheric temperature between winter and summer. The parameterization accounts for the effects of stratospheric temperatures on outgoing IR only to the extent that such temperatures are globally and seasonally correlated with surface temperature. Since the parameterization does not explicitly include stratospheric temperature as a parameter, it does not adequately account for this seasonal influence on outgoing IR. The behavior of the parameterization at 90°S can also be attributed to neglect of stratospheric temperature changes. At 90°S the surface temperatures are −59°C and −51°C in April and October, respectively. Yet, the detailed model gives virtually no difference in outgoing IR. There is too little water vapor in the atmosphere for variations in RH to have much effect. However, the lower stratosphere is considerably warmer in April than in October, thus balancing the decrease in surface contribution. The parameterization follows T_s only and generates an outgoing IR too low in April.

4. Parameterization of cloud effect

Ramanathan (1977) demonstrated that c_2, the cloud modification function in (3), appeared to be a function only of the temperature difference between the surface and the cloud top ($T_{sc} \equiv T_s - T_c$). He used his IR radiative transfer model with the McClatchey *et al.* model atmospheres and a single overcast "black" cloud layer to compute c_2 for various cloud heights. His result was that, to a good approximation,

$$c_2 \text{ [W m}^{-2}] = 1.65 T_{sc}, \tag{6}$$

where T_{sc} is in kelvins. The simplicity of (6) makes it very useful for simple climate models or back-of-the-envelope energy balance diagnostics. The linearity of (6) also facilitates the combination of multilevel clouds into a single effective cloud layer. Even so, an accurate estimation of T_{sc} in terms of a single effective black cloud layer is subject to uncertainty (e.g., Cess and Ramanathan, 1978). Most low clouds are sufficiently thick that they have emissivity close to 1.0, but cirrus clouds are quite variable in thickness and often have emissivity considerably less than 1.0. The corresponding effective black cloud would be lower than the true cirrus altitude or would have reduced fractional coverage. Regardless of this difficulty, (6) does represent a substantial improvement in flexibility and physical comprehensiveness over the empirical forms mentioned earlier in which c_2 is constant or even included in c_1.

During the course of our parameterization development it became apparent that (6) omits a significant factor influencing c_2. We have already shown that a measure of tropospheric relative humidity should be included in the parameterization of clear-sky outgoing IR. It is curious then that an atmospheric opacity parameter like $\widehat{RH}$ does not appear in Ramanathan's calculations of c_2. For example, a

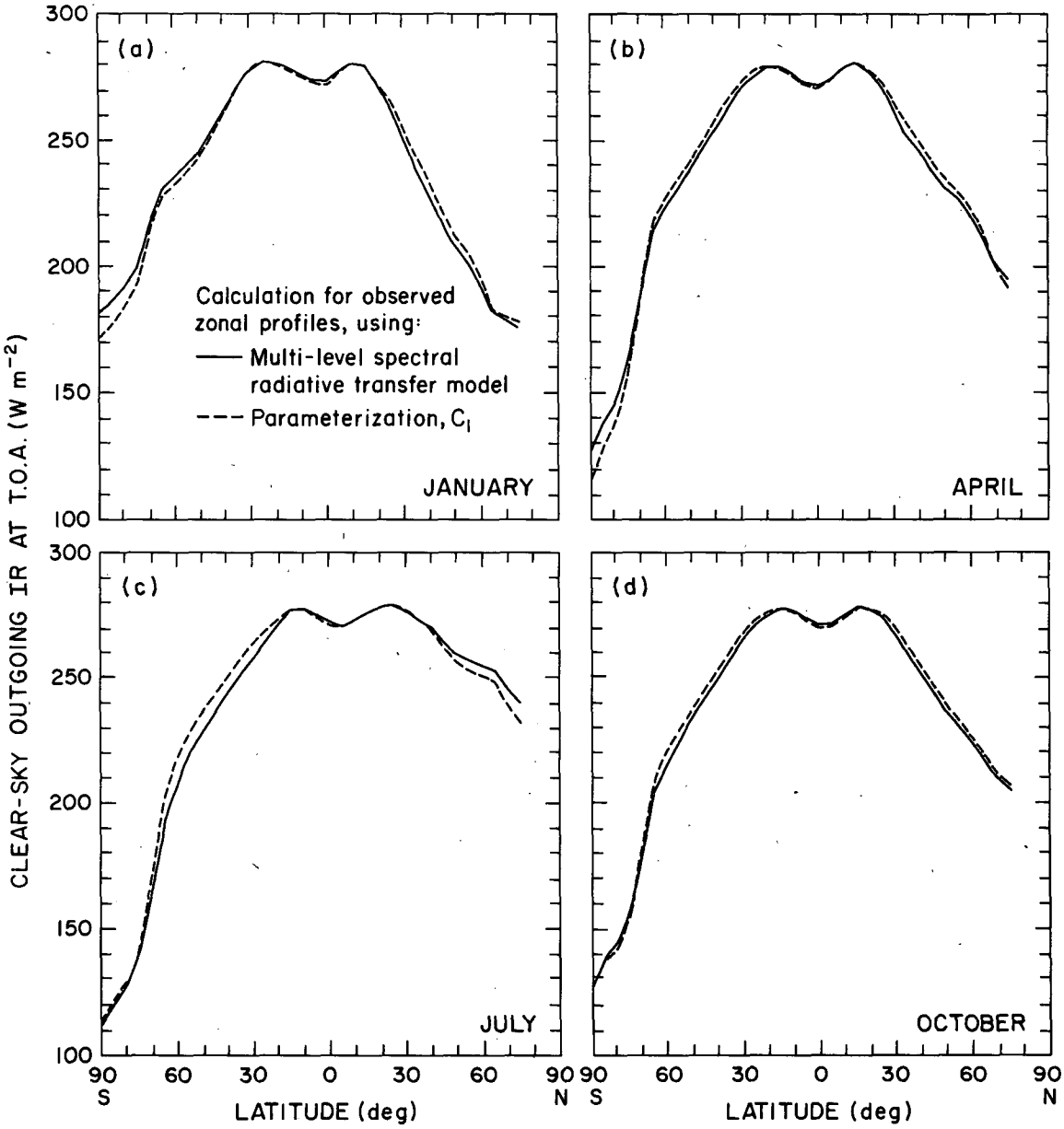

FIG. 10. Comparison of the clear-sky outgoing IR parameterization and the detailed model results for the four mid-season months. Calculations use observed zonal profiles of temperature and humidity (see text).

cloud inserted in a transparent atmosphere will cause F to decrease (i.e., $c_2 > 0$) if $T_c < T_s$, but in an opaque atmosphere a cloud will have no effect on F (i.e., $c_2 = 0$). To resolve this question we have used the detailed model to repeat Ramanathan's calculations with an overcast "black" cloud inserted in the McClatchey temperature profiles, but with several uniformly perturbed RH(z) profiles. Given the temperature profiles, we insert a single "black" cloud layer in the troposphere at the height necessary to achieve the desired T_{sc}.

In Fig. 11 we show c_2 plotted against T_{sc} for the five McClatchey atmospheres, each having tropospheric RH(z) perturbed to give $\widehat{\mathrm{RH}} = 40\%$. As expected all points plot near a straight line with a slight upward concavity, as Ramanathan (1977) found. [Neither the best-fitting straight line nor the apparent trend of the points passes through the origin. This feature is due to our assumption of 0.95 for surface emissivity which causes a black cloud at the surface actually to increase F.] The slope of the plotted line is 1.87 W m^{-2} K^{-1} which is somewhat larger than Ramanathan's value of 1.65 W m^{-2} K^{-1}.

The close match for all five profiles applies only for $\widehat{\mathrm{RH}} = 40\%$, however. If we were to plot all the c_2 points for other values of $\widehat{\mathrm{RH}}$ the scatter about a

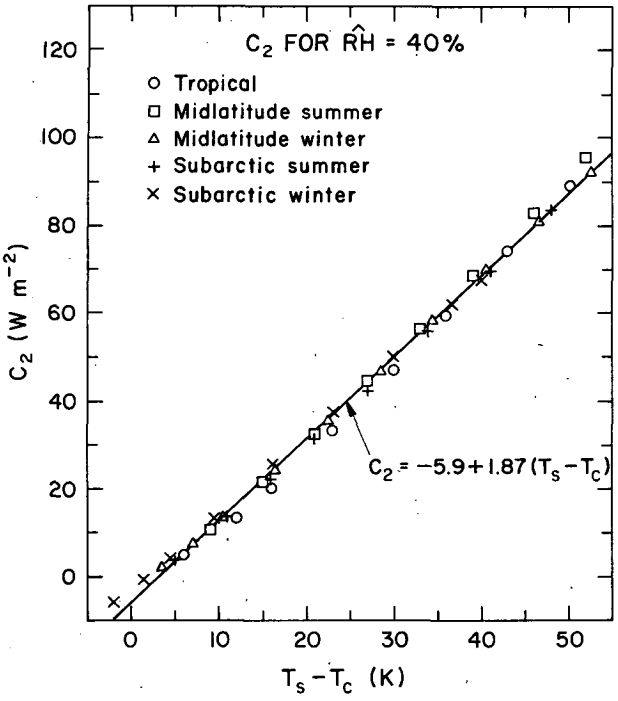

FIG. 11. The cloud modification parameter c_2 as a function of the difference between surface temperature (T_s) and cloud-top temperature (T_c). All the McClatchey *et al.* (1972) model atmospheres are perturbed to give $\widehat{\mathrm{RH}}$ = 40%. The straight line is a linear least-squares fit to the points.

line would increase greatly. As an example we show c_2 for $\widehat{\mathrm{RH}}$ = 30, 60 and 100% for the Tropical temperature profile only in Fig. 12. There is a large difference in c_2 depending on the value of $\widehat{\mathrm{RH}}$ employed. At T_{sc} = 40 K, c_2 varies from 51 W m^{-2} to 72 W m^{-2} for $\widehat{\mathrm{RH}}$ = 100% and 30%, respectively. The c_2 points clearly define a family of curves with smaller c_2 values associated with larger $\widehat{\mathrm{RH}}$, just as intuition would indicate. The Tropical profile (used in Fig. 12) gives the largest change of c_2 with $\widehat{\mathrm{RH}}$; the colder profiles show a much smaller sensitivity.

We can now explain Ramanathan's result for c_2. Each of the McClatchey model atmospheres has RH in the range of 40–60% with higher RH for the colder profiles (Fig. 1). Since the warm profiles have RH near 40% (Tropical and Mid-Latitude Summer) we would expect these to have similar c_2 curves. The colder profiles have $\widehat{\mathrm{RH}}$ of 50–60%, but since $\partial c_2/\partial\widehat{\mathrm{RH}}$ for these profiles is relatively small, their c_2 curves are similar to those of the warm profiles. Apparently for these reasons Ramanathan's calculations did not highlight the effect of variable relative humidity on c_2. In comparing Figs. 1 and 7 we note that the value of $\widehat{\mathrm{RH}}$ of ~0.4 given by the McClatchey *et al.* Tropical atmosphere appears to be low relative to our composite climatology. If the Tropical model atmosphere had an $\widehat{\mathrm{RH}}$ of ~0.55, then Ramanathan's results would have shown the Tropical c_2 values falling away from the line defined by the c_2 values of the other model atmospheres.

Instead of attempting to modify (6) to capture the effect of variable $\widehat{\mathrm{RH}}$ we parameterize c_2 using the parameterization for c_1. We assume that the outgoing IR irradiance from a cloudy area [c_2' in Eq. (4)] can be approximated by c_1 (Table 3) with T_c replacing T_s. Of course, this is only a first approximation since c_1 is derived for a surface emissivity ϵ = 0.95 (the clouds are assumed to have ϵ = 1.0) and for a fixed surface pressure of 1013 mb. We therefore add a term to correct for most of the error made in assuming $c_1(T_c, \widehat{\mathrm{RH}}) = c_2'$. Thus

$$c_2' = c_1(T_c, \widehat{\mathrm{RH}}) - f(T_{sc}, \widehat{\mathrm{RH}}), \tag{7a}$$

or

$$c_2 = c_1(T_s, \widehat{\mathrm{RH}}) - c_1(T_c, \widehat{\mathrm{RH}}) + f(T_{sc}, \widehat{\mathrm{RH}}), \tag{7b}$$

where $f(T_{sc}, \widehat{\mathrm{RH}})$ is the correction term. The parameterization for f, developed using c_2 values computed for the McClatchey $T(z)$ profiles, is given in Table 4. The correction is separated into terms f_1 and f_2, where the former accounts for cloud height and emissivity effects for a standard $\widehat{\mathrm{RH}}$ = 40% and is a function of T_{sc} only; and the latter accounts for $\widehat{\mathrm{RH}}$

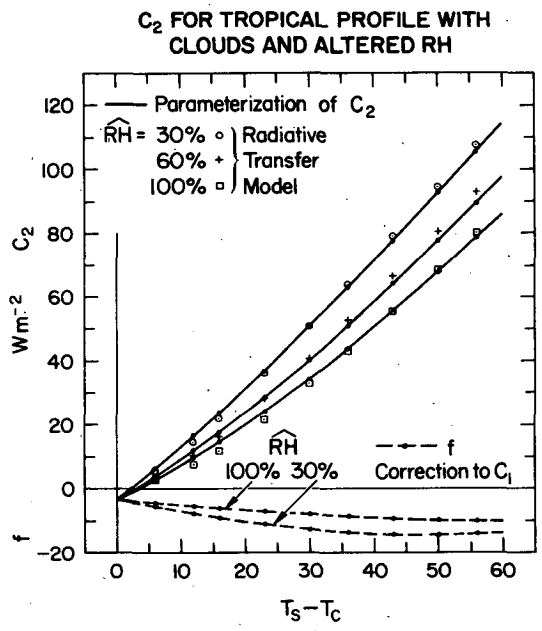

FIG. 12. The cloud modification parameter c_2 (upper three plots) as a function of $T_s - T_c$ and $\widehat{\mathrm{RH}}$ for the McClatchey *et al.* (1972) Tropical temperature profile. The parameterization for c_2 is that given in Table 4. The plots at the bottom are of $f(T_{sc}, \widehat{\mathrm{RH}})$, the correction applied to the clear-sky outgoing IR irradiance parameterization enabling the c_1 functional form to be used to determine c_2 [Eq. (7)].

deviations from a standard $\widehat{\mathrm{RH}} = 40\%$. [Estimates of climatological values of T_{sc} derived from London (1957) for January and July are shown in Fig. 5 of Warren and Thompson (1983).] The parameterization of f is not discussed or criticized in detail here because it is of secondary importance, i.e., $|f| \ll c_1$ in (7).

The parameterization for c_2 is compared to the detailed model results for the Tropical temperature profile in Fig. 12. The agreement is quite good; the rms error for the points shown is 1.7 W m^{-2}. Comparable errors occur for the other four McClatchey temperature profiles. Analysis shows the residual error to be associated with the detailed shape of the RH(z) profiles and is therefore uncorrectable in this parameterization. Also shown in Fig. 12 is the correction term $f(T_{sc}, \widehat{\mathrm{RH}})$ for $\widehat{\mathrm{RH}} = 30\%$ and 100%. Although the correction is generally small compared to $c_1(T_c, \widehat{\mathrm{RH}})$, omitting f would create an error of 3–15 W m^{-2}. Note that f tends to -3.1 W m^{-2} as T_{sc} approaches zero which approximately accounts for the effect of different assumed emissivities of the surface and clouds.

5. Conclusion

The radiative model-based parameterization we have proposed has several advantages over the currently available empirical parameterizations mentioned in the Introduction. First, we have extended the estimated range of validity of the parameterization considerably beyond present climatological limits. Such a feature is important for modeling climatic states far from the present but impossible to incorporate in a parameterization based solely on available observations. Notably, our parameterization of the clear-sky component of outgoing IR is rather linear in T_s over the present range of T_s and $\widehat{\mathrm{RH}}$. This agrees with some empirical studies (e.g., Fig. 5 of Warren and Schneider, 1979).

Second, the parameterization explicitly includes tropospheric relative humidity, cloud-top temperature and cloud amount (for the "effective black cloud") as parameters. This results in substantial flexibility and should facilitate interesting sensitivity studies using vertically averaged models. For example, Barron *et al.* (1981) have already used Ramanathan's (1977) form for c_2 [Eq. (6)] to estimate the sensitivity of the Cretaceous climate to plausible variations in $T_s - T_c$. The explicit separation of tropospheric humidity and cloud effects would be difficult to obtain empirically due to the correlation between these quantities. However, since the correlation between humidity and cloudiness is far from perfect (e.g., Fig. 2 of Slingo, 1980) inclusion of $\widehat{\mathrm{RH}}$ as a parameter potentially allows one to correctly account for changes in relative humidity not associated with cloud changes.

TABLE 4. Parameterization of cloud modification term for outgoing IR irradiance at the top of the atmosphere (z = 50 km).

c_2A_c = cloud modification term (W m^{-2}) [see Eq. (3)].
A_c = fractional total cloud amount.
$T_{sc} = T_s - T_c$ ($0 \leq T_{sc} < 60$ K).
T_s = surface air temperature ($-118°\mathrm{C} \leq T_c \leq 57°\mathrm{C}$).
T_c = cloud top temperature ($-118°\mathrm{C} \leq T_c \leq 57°\mathrm{C}$).
$\widehat{\mathrm{RH}}$ = height-mean relative humidity [Eq. (5)] ($0.2 \leq \widehat{\mathrm{RH}} \leq 1.0$).

PARAMETERIZATION:

$$c_2 = c_1(T_s, \widehat{\mathrm{RH}}) - c_1(T_c, \widehat{\mathrm{RH}}) + f(T_{sc}, \widehat{\mathrm{RH}})$$
(for c_1 see Table 3)
$$f(T_{sc}, \widehat{\mathrm{RH}}) = f_1(T_{sc}) + f_2(T_{sc}, \widehat{\mathrm{RH}})$$
$$f_1 = d_0 + d_1T_{sc} + d_2T_{sc}^2$$
$$f_2 = d_3T_{sc}(T_{sc} + d_4)(\widehat{\mathrm{RH}} + d_5)$$
$$d_0 = -3.1$$
$$d_1 = -0.4146$$
$$d_2 = 4.084 \times 10^{-3}$$
$$d_3 = -4.44 \times 10^{-3}$$
$$d_4 = -80.0$$
$$d_5 = -0.40$$

Warren and Thompson (1983) compare results of the detailed radiative-transfer model to satellite observations of total outgoing IR radiation. However, we have not compared our parameterization of F to satellite observations because, as Warren and Thompson point out, our knowledge of T_c is too uncertain to make such a comparison a strict validation. Indeed, for purposes of modeling climatic sensitivity we recommend tuning the parameterization by choosing T_{sc} so as to match observations of F.

We should point out a significant conceptual difference between our modeling approach and the empirical approach to parameterizing F. The empirical approach attempts to relate spatial and/or temporal means of F to climatological mean parameters. We, however, have essentially related a point value of F (in space and time) to parameters characterizing a single climatological mean atmosphere/surface state. Thus the empirical approach implicitly includes the effects of spatial and temporal variability on the mean F. The effects of unresolved parameter variability can be significant as demonstrated by Warren and Thompson (1982) for $\widehat{\mathrm{RH}}$. Although such variability effects are not built into our parameterization they can be simulated by calculating a mean F for an ensemble of sets of T_s, T_c and $\widehat{\mathrm{RH}}$ having a specified climatological variability.

Beyond any purely practical uses this work should serve as an aid to understanding the dominant factors affecting outgoing infrared irradiance and may help to guide improved semi-empirical parameterizations as better climatological data become available.

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ADDENDUM AND CORRIGENDUM

"Parameterization of Outgoing Infrared Radiation Derived from Detailed Radiative Calculations"

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Our recent paper (Thompson and Warren, 1982) advocated the utility of relative humidity (RH) as a predictor of outgoing infrared radiation, but neglected to state how we computed RH.

We define relative humidity RH as

$$RH = \begin{cases} e/e_{sw}, & T \geqslant 0°\mathrm{C} \\ e/e_{si}, & T < 0°\mathrm{C}, \end{cases}$$

where e is the atmospheric water vapor pressure, e_{sw} is the saturation vapor pressure over liquid water, and e_{si} is the saturation vapor pressure over ice. Our parameterization for outgoing infrared radiation is valid only if this definition of RH is used.

For the calculation of e_{sw} (in millibars) as a function of absolute temperature T, we use the polynomial of Richards (1971) which was popularized in the meteorological literature by Wigley (1974):

$$e_{sw} = 1013.25 \exp(13.3185t - 1.9760\, t^2 - 0.6445t^3 - 0.1299\, t^4),$$

where $t \equiv 1 - 373.15T^{-1}$. For calculations of e_{si} (in millibars) we use the formula of Goff and Gratch (p. 350 of List, 1949):

$$\log_{10}e_{si} = -9.09718(x - 1) - 3.56654 \log_{10}x + 0.876793(1 - x^{-1}) + \log_{10}6.1071,$$

where $x \equiv 273.16T^{-1}$.

Since liquid water droplets can exist in clouds at temperatures below 0°C, our choice of 0°C as the transition temperature is somewhat arbitrary. We thank Dr. Lee Young (AVCO Everett Research Laboratory) for alerting us to this potential source of confusion.

On a related subject, one point in Fig. 1 of Thompson and Warren (1982) was plotted improperly. The McClatchey tropical atmosphere has RH = 68% at 17 km height. It was incorrectly plotted as 78% due to a drafting error. McClatchey *et al.* (1972, Table 1) gave water vapor density at 1 km intervals up to 25 km height. Our figure plotted the corresponding point values of RH and joined them by straight lines, so for the tropical atmosphere our interpolation between 16 and 18 km was drafted incorrectly.

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