SHORT COMMUNICATION

COMMENTS ON "SPECTRAL ALBEDOS OF MID-LATITUDE SNOWPACKS"

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ABSTRACT

Choudhury (1982) has found that small amounts of soot added to a thin snow layer could increase the snowpack albedo in his computer model. This result is physically unrealistic; the effect of adding soot to snow is to darken it, not to brighten it. An explanation for the error in Choudhury’s model is that in his single-scattering computation he has apparently exaggerated the extinction cross-section of soot by eight orders of magnitude.

Carbon-black has been added to snow by researchers several times in the past to enhance the melting rate of snow (e.g. Meiman, 1973), with the hope that this might be an economical method of uncovering grass for early forage by grazing animals (Regelin and Wallmo, 1975). On the other hand, ski-resort managers have the opposite goal: they could use a snow additive to retard the melting rate of thin snowpacks in order to lengthen the ski season. Choudhury (1982) now finds carbon soot to be so versatile that it may serve both purposes.

Choudhury (1982, Fig. 6) has modeled the following situation: a thin layer of coarse-grained snow, only 2 cm thick, on top of wet soil. If the snow is pure, the calculated albedo of the snow—soil system at the blue-green wavelength $\lambda = 0.45 \, \mu m$, is 53%. He finds that the addition of a minute amount of soot, just one part in $10^{10}$ by weight, causes the snow to brighten dramatically in his calculation: the albedo jumps to 97%. Further addition of soot then brings the albedo back down.

To check these results, we have computed the snow albedo using the Mie-scattering and delta-Eddington radiative transfer method of Wiscombe and Warren (1980; hereafter WWI), and Warren and Wiscombe (1980; hereafter WII), and we obtain no such effect. Soot always acts to decrease visible snow albedo, for any snow thickness and any soot content.

The reason for the strange behavior of Choudhury’s model is traced to his calculation that a 20-mm layer of coarse-grained snow (grain radius $r = 1000 \, \mu m$, density $\rho = 350 \, kg \, m^{-3}$) becomes optically semi-infinite (i.e. that its flux-transmissivity for visible light goes to zero) when one part soot in $10^{10}$ ($r = 0.1 \, \mu m$, $\rho = 1130 \, kg \, m^{-3}$) is added to the snow. By contrast, we calculate that the flux-transmissivity is essentially unaltered when such a tiny amount of soot is added: at visible wavelength 0.45 $\mu m$, about half of the light incident on the snow layer is transmitted to the soil below, just as it is for pure snow.

To investigate this discrepancy, we have tried to duplicate Choudhury’s results using his own equations for the single layer case illustrated in his Fig. 6. We take Choudhury’s specifications for his “shallow snowpack” which we work through in detail for the visible wavelength $\lambda = 0.45 \, \mu m$. From Mie calculations, the single-scattering quantities we derive are given in Table 1, where $Q_{ext}$ is the sum of the absorption efficiency, $Q_{abs}$, and the scattering efficiency, $Q_{sca}$.

<table>
<thead>
<tr>
<th>ice, $r = 1000 , \mu m$</th>
<th>soot, $r = 0.1 , \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>extinction efficiency, $Q_{ext}$</td>
<td>2.003</td>
</tr>
<tr>
<td>absorption efficiency, $Q_{abs}$</td>
<td>8.595 x 10^{-5}</td>
</tr>
<tr>
<td>asymmetry parameter, $G$</td>
<td>0.890</td>
</tr>
</tbody>
</table>

Although Choudhury does not state explicitly how he derives the extinction coefficient $\gamma_e$ (units m$^{-1}$), we can assume it is obtained in the usual way as

$$\gamma_e = \gamma_e(\text{ice}) + \gamma_e(\text{soot}),$$

(1)
where
\[ \gamma_e(\text{ice}) = Q_{\text{ext}}(\text{ice}) A_{\text{ice}} N_{\text{ice}} \]
and
\[ \gamma_e(\text{soot}) = Q_{\text{ext}}(\text{soot}) A_{\text{soot}} N_{\text{soot}}. \]

(2)

The quantities needed are the cross-sectional area of a single ice sphere \( A_{\text{ice}} = \pi 10^{-6} \text{ m}^2 \) per particle and a single soot sphere \( A_{\text{soot}} = \pi 10^{-14} \text{ m}^2 \) per particle, together with the number of ice particles per unit volume \( N_{\text{ice}} \) and soot particles per unit volume \( N_{\text{soot}} \). The density of the snowpack, together with the ice particle radius, determines \( N_{\text{ice}} = 9.1 \times 10^7 \text{ particles per m}^3 \). If the weight fraction of soot is \( 10^{-10} \), then \( N_{\text{soot}} = 81 N_{\text{ice}} \). We then obtain

\[ \gamma_e(\text{ice}) = 5.7 \times 10^2 \text{ m}^{-1} \]
and
\[ \gamma_e(\text{soot}) = \begin{cases} 5.8 \times 10^{-4} \text{ m}^{-1} & \text{if soot fraction} = 10^{-10} \\ 5.8 \times 10^{-6} \text{ m}^{-1} & \text{if soot fraction} = 10^{-12}. \end{cases} \]

(3)

The single-scattering albedo (\( \Omega \) in Choudhury’s notation) is given by

\[ \Omega = \gamma_e^{-1} \left[ Q_{\text{ext}}(\text{ice}) A_{\text{ice}} N_{\text{ice}} + Q_{\text{ext}}(\text{soot}) A_{\text{soot}} N_{\text{soot}} \right]. \]

(4)

We can now substitute \( \Omega, G, \) soil albedo, and \( \gamma_e \) into Choudhury’s radiative-transfer equations (4, 7, 10–14 and 19). (However, in order to remove the diffraction component from the scattering by the ice grains as Choudhury has done, when computing \( \gamma_e \) we use not \( Q_{\text{ext}} \) but rather \( Q_{\text{ext}}(\text{effective}) = Q_{\text{ext}} - 1 \). For the case of pure snow, we obtain an albedo of 0.528, in agreement with his Fig. 6. This suggests that his radiative transfer model is consistent with his equations, which in turn appear to be correct.

We next add soot in the amount of one part in \( 10^{10} \), for which Choudhury finds an albedo of 0.97 at 0.45 \( \mu \text{m} \). Our calculation, however, indicates that the albedo remains the same as for pure snow to at least four significant figures. A corroborative result can also be inferred from Fig. 7b of WWII. We are thus unable to confirm the dramatic brightening of the snowpack due to the addition of small quantities of soot.

The spurious albedo is very likely due to an erroneous calculation of \( \gamma_e \) which causes the snowpack to appear optically semi-infinite when only minute quantities of soot are added. Since Choudhury’s albedos of semi-infinite snow appear reasonable, the error must be in \( \gamma_e \) and not also in \( \Omega \). The albedo of semi-infinite snow does not depend on \( \gamma_e \), so an error in its computation would not show up there. It is possible for Choudhury to have made an error in \( \gamma_e \) without having that error also show up in his calculation of \( \Omega \) because he apparently did not use (4) to calculate \( \Omega \) but instead used a dielectric-mixing formula.

We can best estimate the magnitude of the error in \( \gamma_e \) by considering his example of an even smaller amount of soot, one part in \( 10^{12} \) (10\(^{-6}\) ppmw in Choudhury’s notation). Comparing his Fig. 6 to Fig. 13c of WWII, we see that Choudhury’s addition of one part soot in \( 10^{12} \) to a 7 mm-liquid-equivalent snowpack has the same effect as that of doubling the thickness of a pure snowpack from 7 to 14 mm. This indicates that when the soot fraction is \( 10^{-12} \), Choudhury is finding \( \gamma_e(\text{soot}) \approx \gamma_e(\text{ice}) \), instead of the correct result \( \gamma_e(\text{soot}) \approx 10^{-8} \gamma_e(\text{ice}) \) given by (3). We therefore estimate that Choudhury exaggerated \( \gamma_e(\text{soot}) \) by a factor of approximately \( 10^8 \).

One possible way in which this error could arise is if \( A_{\text{soot}} \) were mistakenly equal to \( A_{\text{ice}} \) in (2), since \( A_{\text{soot}}/A_{\text{ice}} = (\tau_{\text{soot}}/\tau_{\text{ice}})^2 = 10^{-8} \).

In summary, we think that Choudhury’s model has a serious flaw which leads to a spurious enhancement of the albedo of an optically thin snowpack when soot is added. The error appears to entail a factor of \( 10^{-8} \) in his computation of the extinction coefficient of the soot particles. This would be the result if he has not properly taken into account the relative cross-sectional areas of soot and ice grains. Although this error is quite serious for thin snow layers, it would not change the results for uniform optically thick snowpacks.

ACKNOWLEDGEMENTS

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REFERENCES

Regelin, W.L. and O.C. Wallmo (1975), Carbon black increases snowmelt and forage availability on deer winter range in Colorado, USDA Forest Service Research Note RM-296, Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colorado.