Migration of air bubbles in ice under a temperature gradient, with application to "Snowball Earth"

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[1] To help characterize the albedo of "sea glaciers" on Snowball Earth, a study of the migration rates of air bubbles in freshwater ice under a temperature gradient was carried out in the laboratory. The migration rates of air bubbles in both natural glacier ice and laboratory-grown ice were measured for temperatures between −36°C and −4°C and for bubble diameters of 23–2000 μm. The glacier ice was sampled from a depth near close-off (74 m) in the JEMS2 ice core from Summit, Greenland. Migration rates were measured by positioning thick sections of ice on a temperature gradient stage mounted on a microscope inside a freezer laboratory. The maximum and minimum migration rates were 5.45 μm h^{-1} (K cm^{-1})$^{-1}$ at −4°C and 0.03 μm h^{-1} (K cm^{-1})$^{-1}$ at −36°C. Besides a strong dependence on temperature, migration rates were found to be proportional to bubble size. We think that this is due to the internal air pressure within the bubbles, which may correlate with time since close-off and therefore with bubble size. Migration rates show no significant dependence on bubble shape. Estimates of migration rates computed as a function of bubble depth within sea glaciers indicate that the rates would be low relative to the predicted sublimation rates, such that the ice surface would not lose its air bubbles to net downward migration. It is therefore unlikely that air bubble migration could outrun the advancing sublimation front, transforming glacial ice to a nearly bubble-free ice type, analogous to low-albedo marine ice.

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1. Introduction

[2] According to the "Snowball Earth" hypothesis, nearly the entire ocean was covered with thick ice at times during the early Paleoproterozoic (2.45–2.22 billion years ago) and the late Neoproterozoic (0.73–0.58 billion years ago) [e.g., *Hoffman and Schrag*, 2002]. This hypothesis was put forward to explain unusual geological and geochemical findings for the Neoproterozoic [*Harland*, 1964; *Kirschvink*, 1992; *Hoffman and Schrag*, 2000]. Tropical glaciation would have been facilitated by a runaway albedo feedback of snow and ice that can occur when oceanic freezing reaches the subtropics [*Budyko*, 1969; *Sellers*, 1969]. The duration of each snowball event is thought to have been several million years [*Hoffman and Schrag*, 2002]. The major events of the Neoproterozoic were the Sturtian and Marinoan glaciations, dated at 716 and 635 million years ago [*Macdonald et al.*, 2010].

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[3] Because of the abundant sunlight available at low latitudes, the nature of possible snowball events depends crucially on the surface albedo of the tropical ocean. Besides determining the ice thickness [*Warren et al.*, 2002], which constrains potential refugia for photosynthetic life, the albedo also determines (1) the drawdown of atmospheric CO_2 necessary to initiate the snowball catastrophe, (2) the critical latitude for the ice-albedo instability, (3) surface temperatures, and (4) the duration of a snowball event (how much volcanic emission of CO_2 is needed to warm the climate and melt the ice).

[4] Despite the name, a "snowball" Earth would not have been completely snow-covered. As is the case today, evaporation (or sublimation) would have exceeded precipitation in large areas of the tropics, although the hydrological cycle was probably weakened by a factor of ~300 [*Pollard and Kasting*, 2004]. The zone of net sublimation would have extended from ~10°S to ~10°N [*Pierrehumbert*, 2005] up to ~30°S to ~30°N [*Pollard and Kasting*, 2004]. If the ocean surface did indeed freeze to the equator, its surface in the equatorial zone would have had transient snow or frost, but more typically would have been bare ice. A variety of ice types, each with different albedo, would have been present at different times and latitudes [*Warren et al.*, 2002]. The initial freezing would have produced sea ice with brine inclusions; cooling would have caused the precipitation of salts, and sublimation would then result in the formation of a

salt lag deposit at the surface. These ice surfaces have been studied in the laboratory by *Light et al.* [2009].

[5] As the ocean lost its reservoir of heat, the ice would have grown until reaching an equilibrium thickness set by the geothermal heat flux and the surface temperature. The resulting thicknesses would have been hundreds of meters, with thicker ice at high latitude than at low latitude. The resulting thickness gradient would have caused the ice to flow [*Goodman and Pierrehumbert*, 2003; *Goodman*, 2006; *Pollard and Kasting*, 2005, 2006]. According to our best understanding, the ocean surface eventually became covered with glacier ice in the form of "sea glaciers" (flowing like modern ice shelves in response to their thickness gradient, but not dependent on continental glaciation). The equilibrium thickness might be 1 km at the pole but only half that at the equator, because of the higher surface temperature at the equator. The sea glaciers are computed to flow as much as 60 m/yr even when they cover the entire ocean [*Goodman*, 2006, Figure 2g]. After the few thousand years required for the ocean to lose its reservoir of heat, the original sea ice of the equatorial ocean would be crushed by the inflow of the much thicker sea glaciers, so the area of the ocean surface covered by salty ice would be mostly replaced by freshwater ice. Because of the flow, the high-latitude ice thickness would be less than the equilibrium value determined by the geothermal heat flux, so seawater would freeze to the base. Correspondingly, at low latitudes the inflow would cause the ice to be too thick for equilibrium, so the frozen seawater would melt off the base [*Goodman*, 2006, Figure 1c]. The tropical ice would lose mass both from the bottom (by melting) and from the top (by sublimation), balanced by inflow of sea glacier ice from higher latitude. These sea glaciers were snow-covered at high latitude, but as they entered the tropics the surface snow and subsurface firn sublimated away, leaving bare freshwater ice. Such ice would have contained numerous bubbles because its origin was compressed snow, and the number, density, and size distribution of these bubbles would have determined the albedo of the ice.

[6] The modern example of bare glacier ice exposed by sublimation, which has never experienced melting, is the "blue ice" areas of Antarctica. The available albedo measurements were reviewed by *Warren and Brandt* [2006]; they are in the range 0.55–0.65. In these regions, the lower albedo (compared to snow), as well as the smaller shortwave irradiance extinction coefficient, cause shortwave radiation to penetrate relatively deep into the ice, leading to significantly higher ice temperatures (by up to 7 K) when compared with nearby snow-covered ground [*Bintanja and van den Broeke*, 1995; *Bøggild et al.*, 1995].

[7] The blue-ice albedos may or may not be representative of sea glacier albedo over the long duration of a snowball event. The equatorial ocean surface would have been continuously renewed by very slow net sublimation of 1–10 mm/yr [*Pollard and Kasting*, 2004; *Pierrehumbert*, 2005], but the bubbles may have been able to migrate downward toward higher temperatures. If the sublimation rate exceeded the migration rate, then the albedo would have remained high, favoring the persistence of the snowball state. If, on the contrary, the migration rate exceeded the sublimation rate, then the albedo would have decreased as light-scattering from bubbles would have occurred deeper within the ice. To evaluate the effect of bubble migration on the albedo, we measured bubble migration in ice under a temperature gradient. We conclude that under almost all possible conditions the bubbles will not outrun the sublimation front, and the albedo will stay high.

2. Temperature Gradient Bubble Migration

[8] When a temperature gradient is applied to bubbly ice, air bubbles normally migrate slowly toward the warmer ice. This motion is caused by sublimation from the warm wall and subsequent frost deposition on the cold wall [e.g., *Nakaya*, 1956; *Stehle*, 1967; *Shreve*, 1967]. The speed of a bubble is linearly proportional to the magnitude of the temperature gradient across the bubble and depends on bubble temperature, bubble pressure, and bubble shape [*Shreve*, 1967]. The rate-limiting processes for air bubble migration are sublimation of the vapor from the warm wall and diffusion of the vapor from the warm wall to the cold wall. The sublimation is governed by the temperature of the surrounding ice, while the vapor diffusion is governed by both the temperature and the air pressure within the bubble. Shape plays a role in that bubbles that are oblate in the direction of the temperature gradient favor sublimation as the rate-limiting process and are therefore faster, while prolate bubbles are diffusion-limited [*Shreve*, 1967].

[9] Previous laboratory measurements of bubble migration rates in laboratory-grown ice [*Stehle*, 1967; *Shreve*, 1967] suggest that for the slow hydrological cycle of Snowball Earth, the migration rates would be much slower than the sublimation rates, so the bubbles would not be able to outrun the sublimation front and the albedo should remain high as new bubbly ice is continuously exposed at the surface. However, the measurements presented by *Stehle* [1967] were mostly for temperatures above −10°C, whereas daytime surface temperatures in the tropics during the early part of a snowball event could have been much colder, near −30°C [*Pollard and Kasting*, 2004, 2005; *Goodman*, 2006].

2.1. Experiments

[10] We extended the measurements of *Stehle* [1967] to temperatures as low as −37°C using both natural glacier ice (where the bubbles were formed by compression of snow) and laboratory-grown ice (where the bubbles were formed by freezing of liquid water, as air came out of solution). The mechanism of air bubble formation by freezing of liquid water has previously been described by *Carte* [1961] and *Maeno* [1967].

[11] The glacier ice used in this study was from 74 m depth in an ice core from Summit, Greenland. The JEMS2 location is close to the location of the GISP2 ice core, for which a detailed description of air bubbles is given by *Gow et al.* [1997]. The atmospheric pressure at Summit fluctuates around 0.7 atm. Mean diameter (d) for the bubbles observed in this work was 310 μm. The ice at 74 m is near the close-off depth.

[12] The laboratory-grown ice samples were produced by freezing liquid water in a small container at two different temperatures, −6°C and −18°C. The container was open to the atmosphere on the top. The atmospheric pressure in the lab was close to 1 atm. The migration experiments were started 1 day after the sample was frozen. All samples

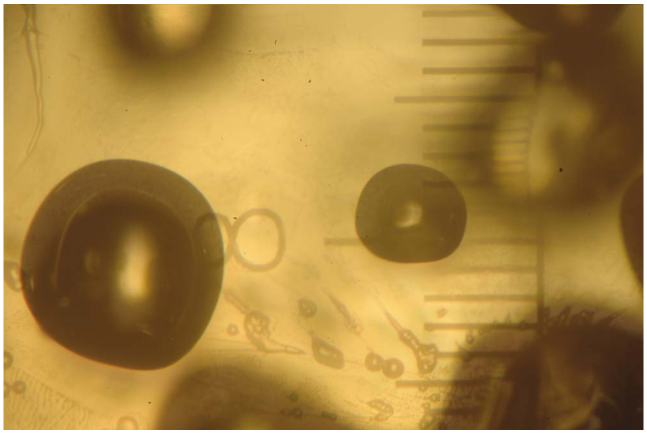

Figure 1. An example of a field of view of air bubbles in ice. A picture of the two bubbles in focus is laid over a picture with the reticle in focus.

contained threads of spherical air bubbles, as well as cylindrical bubbles, aligned parallel to the direction of growth. The bubbles were usually concentrated in planes perpendicular to the growth direction. A large part of the ice block either remained free of bubbles or contained only threads of air or very small bubbles (d < 50 μm). Ice grown more slowly at higher temperatures produced fewer but larger bubbles, as was found earlier by *Carte* [1961]. The bubbles used in the experiments were either spherical or slightly elongated in the direction of ice growth. The mean diameter for the bubbles observed in the block frozen at −6°C was 360 μm; the mean diameter for the bubbles observed in the block frozen at −18°C was 150 μm. Only nearly spherical bubbles were measured in the laboratory-grown ice. We did not measure crystal orientation in any of the samples, but *Stehle* [1967] found that the migration velocity is independent of the angle between the *c* axis and the temperature gradient.

[13] The migration experiments were carried out using the same equipment that *Light et al.* [2009] used to measure brine migration in sea ice. In short, a laboratory microscope was adapted to support a custom-built stage capable of being cooled as much as 15 K below ambient temperature at one end. The microscope was installed in a freezer laboratory, which maintained the room temperature anywhere between −3°C and −30°C. All natural ice samples were cut with their long dimension along the direction of the ice core (originally vertical), so that they could be mounted on the microscope stage in the proper orientation for simulating ice with a warm lower boundary and cold upper boundary. The laboratory-grown ice samples were cut with their long edge perpendicular to the direction of growth, because of the small size of the ice block. The warm end of the sample was mounted on the plate extending into the ambient air of the room; the cold end was mounted to a separate plate in thermal contact with a thermal-electric cooler. A glass reticle was mounted on the underside of the sample; it served as a reference marker for measuring the displacement of individual bubbles during intervals when the temperature gradient was applied. Once mounted on the microscope, a variety of fields could be selected by translating the stage front to back. To estimate the motion of individual bubbles, still photographs were taken of the ice sample in transmitted light both before and after application of the temperature gradient. Figure 1 shows a field of view of migrating bubbles. Migration distances were measured in the images by counting pixels between the reference marker and each end of the bubble using the 'ruler tool' in Adobe Photoshop CS4.0. The migration distance was measured at both the cold end and the warm end of the bubble. We measured distances from ~10 to ~400 μm, or 5 to 300 pixels on the photograph.

[14] Temperature gradients found in nature, particularly in thick ice, would be considerably smaller than the large gradients imposed in this laboratory experiment. Such large gradients were necessary to obtain measurable displacement over experimentally feasible time intervals (hours to weeks). In their studies, *Stehle* [1967] and *Shreve* [1967] addressed the scalability of temperature gradients and found that migration velocities could be normalized by the temperature gradient. We confirmed this by applying different temperature gradients (0.9–4.4 K/cm) for migration experiments at ~−20°C. For most other experiments, we imposed temperature gradients of ~5 K/cm. Temperature gradients as small as 1 K/cm were necessary to measure migration rates at high temperatures without melting the samples. The duration of the experiments was between 5 and 70 h.

2.2. Migration Results and Comparison With Theory

[15] Observed air bubble migration rates, in both glacier ice and laboratory-grown ice (Figures 2 and 3), are comparable to those measured by *Stehle* [1967], and generally agree with theory [*Shreve*, 1967] for air pressures between 1 and 10 atm. There appears to be no difference in migration rate between glacier ice and laboratory-grown ice other than the differences caused by different bubble pressures. Theoretically, the migration rate is slower for bubbles with higher air pressure, because the air impedes diffusion of water vapor. In glacier ice, we measured the migration of different-sized as well as different-shaped bubbles (Figure 2). In contrast to the theory and to previous measurements, we found air bubble migration rates to be proportional to bubble size. The apparent dependence on bubble size is most likely a dependence on bubble pressure. As will be discussed below, the smaller bubbles probably closed off earlier and attained higher pressures by the time of coring.

[16] Despite the observations of *Stehle* [1967] and *Shreve* [1967], the large temperature gradients imposed in the laboratory did appear to influence the migration rates. Bubbles that were observed to have high migration rates accumulated frost. This frosting is due to rapid deposition of water vapor at the cold end. Such rapid frost deposition at the cold end leads to irregular surfaces and to an apparent elongation of the bubble in the direction of the temperature gradient (Figure 4). In the cases of such bubble elongation, we assumed that the more representative migration rate was at the warm end, because its smooth surface could be more accurately defined. Also, as the cold end migrates irregularly, it can leave behind tiny bubbles that become separated from the main bubble. For longer experiments with large temperature gradients, the bubbles filled with frost. Bubble elongation in the direction of migration as well as frosting and bubble separation was previously discussed by *Stehle* [1967].

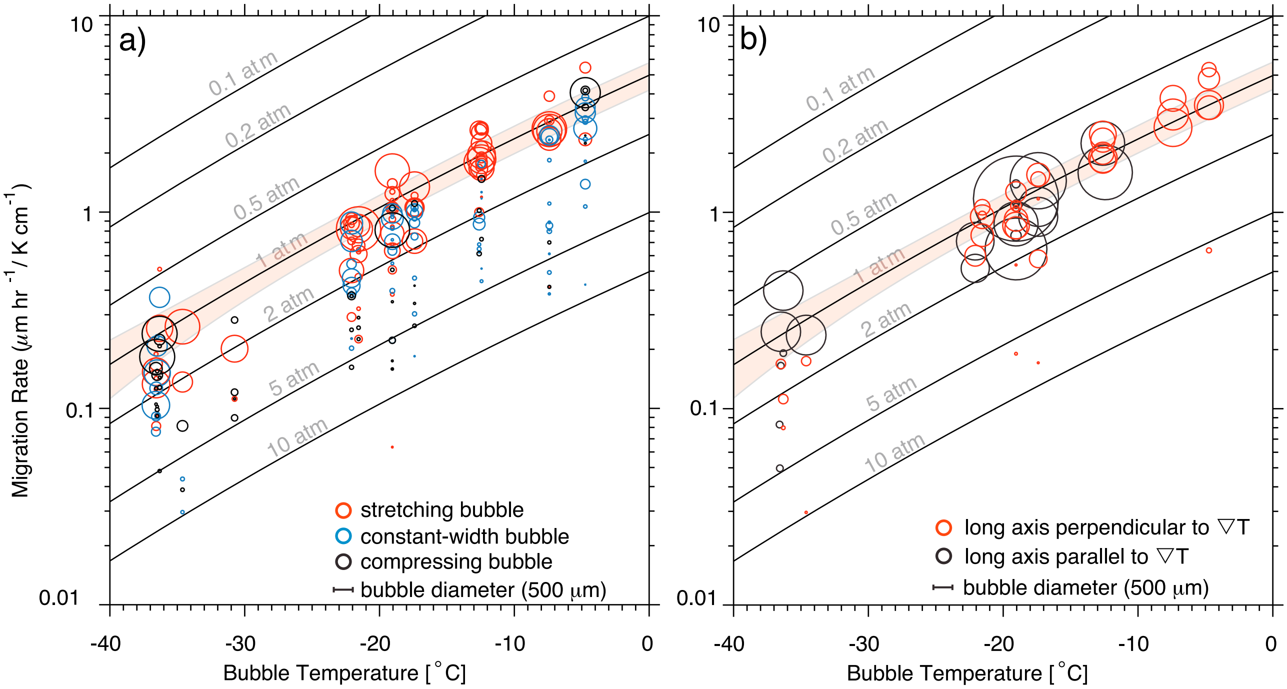

Figure 2. Migration of air bubbles as a function of temperature in glacier ice. Bubble size is defined as the size in the direction of the temperature gradient (∇T), before the migration experiment, and is proportional to circle radii. Migration of widening bubbles (in direction of ∇T) is measured at the faster, warm end; migration of thinning bubbles (in direction of ∇T) is measured at the faster, cold end; and migration of constant-width bubbles is an average between both ends. Indicated in gray are the theoretical curves for migration of spherical air bubbles at different pressures [*Shreve*, 1967]. The shaded red band is the estimated measurement uncertainty for bubbles at 1 atm. (a) Nearly spherical bubbles, with short and long diameter differing by <25%. (b) Nonspherical bubbles with the long axis perpendicular to (red) or parallel to (black) the applied temperature gradient.

[17] We also observed that instead of elongating, some bubbles shortened in the direction of the temperature gradient. This appears to be caused by an interaction between the sublimation front and some sort of barrier, such as a grain boundary, as illustrated in Figure 5 and presented in Figures 2a and 3 ("compressing bubble"). In cases of such shortening, we assumed the migration at the faster cold end to be more representative. For bubbles that experienced differences of <30% in migration rates at the two ends, we report the average of the two rates (Figure 2a, blue circles). Elongation of bubbles parallel to the temperature gradient mainly occurs in larger and faster bubbles (Figures 2a and 3, red circles).

[18] *Shreve* [1967] reported a dependence of migration rate on bubble shape, but our data (Figure 2) do not show such a dependence. We define a "spherical" bubble as one whose long and short axes differ by <25% (Figure 2a). For "nonspherical" bubbles, with difference >25%, the bubbles are classified according to whether their longer axis was perpendicular or parallel to the temperature gradient (Figure 2b). The nonspherical bubbles are generally larger than spherical bubbles. The migration rates of these bubbles (Figure 2b) exhibit less scatter than the migration rates of the smaller, spherical bubbles (Figure 2a) and they scatter around the theoretical migration values for pressures near 1 atm. There was no significant difference in migration rates between bubbles with their long axis perpendicular to the temperature gradient and bubbles with their long axis parallel to the temperature gradient.

[19] *Stehle* [1967] attributed the observed scatter in migration rates to uncontrolled variations in pressure, and possibly shape as well. We do not have pressure measurements for any of our measured bubbles. While we cannot estimate the internal pressure of bubbles in glacier ice, we can assume that the bubbles in our laboratory-grown ice, which were all measured in a single experiment and which were all about the same size, would also all have approximately the same pressure and should therefore theoretically have the same migration rate (Figure 3). To get an idea of measurement uncertainties, we grouped bubbles of the same size in each experiment and calculated the standard deviation of each group (shaded red band centered on the "1 atm" line in Figures 2 and 3). The relative standard deviation increases with decreasing migration rates (i.e., with higher pressure or lower temperature).

3. Discussion

[20] We measured the migration of air bubbles in natural and laboratory-grown ice for temperatures between −36°C and −4°C, and for bubble diameters 23–2000 μm. The migration rates range from 0.03 to 5.45 μm h^{-1} (K cm^{-1})$^{-1}$, faster

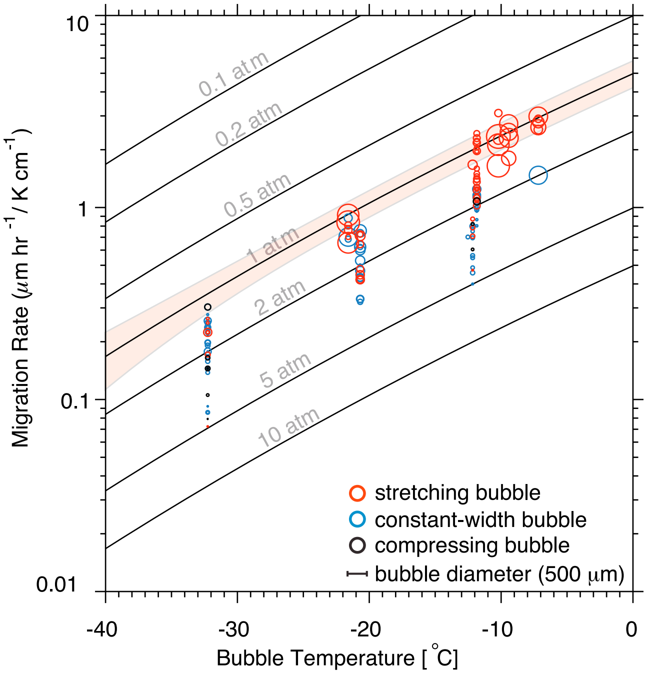

Figure 3. Migration of air bubbles as a function of temperature in laboratory-grown ice. Bubble size is defined as the size in the direction of the temperature gradient (∇T), before the migration experiment, and is proportional to circle radii. Migration of widening bubbles (in direction of ∇T) is measured at the faster, warm end; migration of thinning bubbles (in direction of ∇T) is measured at the cold end; and migration of constant-width bubbles is an average between both ends. Indicated in gray are the theoretical curves for migration of spherical air bubbles at different pressures [*Shreve*, 1967]. The shaded red band is the estimated measurement uncertainty for bubbles at 1 atm.

at high temperatures and for larger bubbles. In contrast to the theory and to previous measurements, we found that air bubble migration increases with bubble size (Figure 6). However, this dependence is probably not a true effect of bubble size, but more likely reflects the distribution of bubble pressures at the depth of the ice core sample. The bubble pressure at close-off corresponds to the atmospheric pressure at the altitude of the ice surface (approximately 0.7 atm at Summit, Greenland). After close-off, ice density increases primarily in response to hydrostatic compression of the trapped air bubbles, and the bubbles shrink until their pressure is equal to the hydrostatic pressure [e.g., *Jones and Johari*, 1977]. Equalization of the air bubble pressures with the ice overburden pressure occurs at about 300 m at GISP2 [*Gow et al.*, 1997]. At shallower depths, the bubble pressure lags the overburden pressure by up to 4–5 atm (approximately the difference between atmospheric pressure at close-off depth and ice overburden pressure) [*Langway*, 1958; *Gow*, 1968]. According to *Langway* [1958], and *Gow* [1968, 1971], bubbles with pressures <10 atm do not undergo significant decompression after the ice core is exposed to atmospheric pressure, so we assume that the bubble pressure in the measured bubbles corresponds to the pressure the ice would have had at depth before coring. Bubbles that closed off closer to the surface are therefore smaller, and their pressure increases with increasing depth [e.g., *Gow*, 1970]. The close-off depth of single bubbles can vary by tens of meters, which leads to a distribution of pressures and sizes for bubbles at a given depth. *Lipenkov* [2000] identified two generations of bubbles in the Vostok ice core. Besides the bubbles formed in the close-off zone, a significant number of very small bubbles, which were formed by sublimation/condensation processes in shallower sections of the firn, were identified. *Lipenkov* [2000] shows that these microbubbles have a higher pressure than the larger bubbles (those that do not exceed 2 atm), which agrees with our bubble migration measurements in which small bubbles are slower than larger bubbles (Figure 2). Theoretically, the bubble pressure for our samples can range from atmospheric pressure for bubbles that were just closed off (0.7 atm), to maximal 5.8 atm for bubbles in equilibrium with the overburden pressure at 75 m depth [e.g., *Maeno*, 1982; *Gow et al.*, 1997]. At Summit (GISP2) the firn-ice transition (pore close-off) was reached at 75–77 m at a density of 830 kg m^{-3} and an ice overburden pressure of 5.0–5.1 bars [*Gow et al.*, 1997].

[21] For completeness, we consider whether the air pressure and the associated migration rate in bubbles of different sizes might be affected by the differences in surface energy (surface tension) of bubbles due to differences in curvature (1/radius) [*Ketcham and Hobbs*, 1969]. The surface energy of a bubble adds to the air pressure inside the bubble. For example, a bubble with a radius of 500 μm has a surface

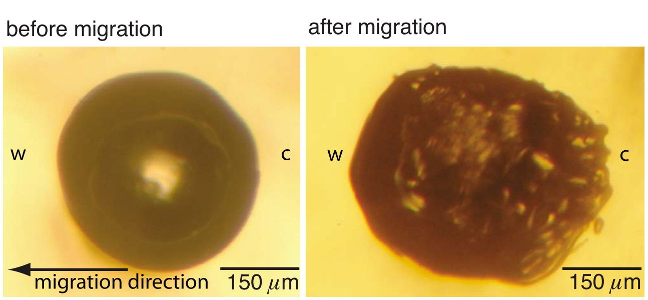

Figure 4. Glacier ice bubble (left) before and (right) after migration. The bubble was at −12.4°C and migrated under a temperature gradient of 5.0 K/cm for 15.4 h. The migration at the warm left end was 2.5 times that at the cold end. After migration, the bubble shows an irregular surface and frosting on the cold right wall as well as hints of bubble separation as the cold end migrates.

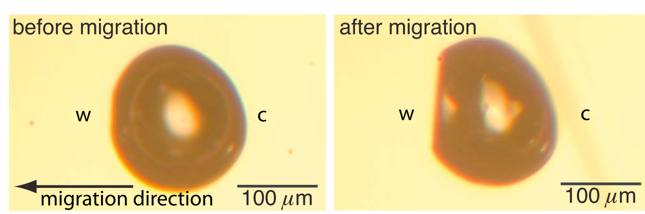

Figure 5. Glacier ice bubble (left) before and (right) after migration. The bubble was at −4.8°C and migrated at a temperature gradient of 1.3 K/cm for 5.5 h. The migration at the warm left end was only 44% of the migration at the cold end. The warm end seems to have encountered a barrier, which lowered its migration rate.

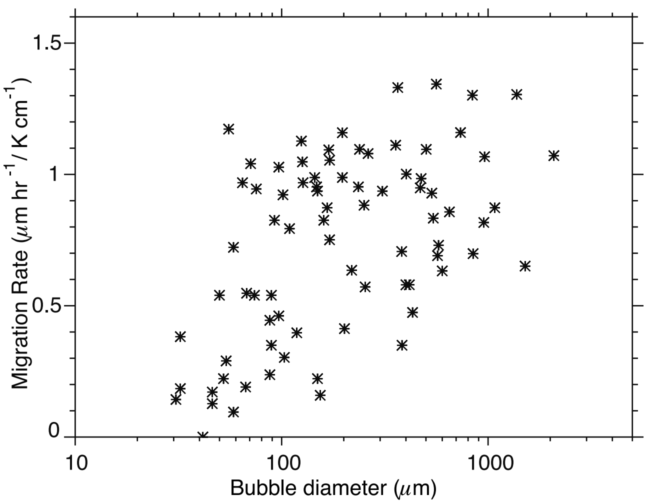

Figure 6. Air bubble migration as a function of bubble diameter for temperatures between −17.4°C and −19.1°C.

energy of 0.004 atm, and a bubble with a radius of 50 μm has a surface energy of 0.043 atm. The resulting pressure difference between these two bubbles is 0.039 atm, too small to explain the pressure variations implied by theory and measurements (Figure 2a), and very small compared to the expected spread of pressures due to close-off timing.

[22] As noted above, we did not see significant differences in migration rates between spherical and nonspherical bubbles of similar size. Nonspherical bubbles are generally larger than spherical bubbles, which suggests that the nonspherical bubbles have been closed off only recently and have not had time to adapt to the hydrostatic pressure at given depth, which would lead to bubble shrinking and rounding. We therefore expect the larger, nonspherical bubbles to have lower pressures. The migration rates of these bubbles (Figure 2b) are less scattered than the migration rates of the smaller, spherical bubbles (Figure 2a) and they scatter around the theoretical migration rates for pressures around 1 atm. The pressure of the open pores would be about 0.7 atm, and we expect the recently closed bubbles to have slightly higher pressures.

[23] We believe that the frosting in bubbles (Figure 4) is caused by the large experimental migration rates, which in turn are caused by large temperature gradients in the experiment that would not be experienced in nature. The assumption that the vapor density near the bubble walls is approximately the static equilibrium density [*Shreve*, 1967] requires that the vapor flux in the bubble is sufficiently small. This assumption seems not to be valid for rapid migration rates where the vapor density is too high for given temperature, which leads to frost precipitation on the cold bubble wall [*Tiller*, 1963]. The cold end appears to migrate more slowly than the warm end, because the frost on the cold wall does not form a clear boundary. The bubble therefore appears to elongate in the direction of the temperature gradient. The frost deposition increases with time, and the movement of the cold wall appears to be slower than the movement at the warm end. After measuring migration in two bubbles after complete frosting, *Stehle* [1967] noted that the effect of frosting on migration rates is not apparent. Those measurements show that the migration rates at first increased during the first 19 h after being frosted, but thereafter decreased [*Stehle*, 1967, Figure 8]. The presence of frost increases the effective thermal conductivity of the bubble [*Shreve*, 1967]; heat is conducted faster through the frosted parts of the bubble, causing a larger temperature gradient through the air, because the distance between the frost and the warm end is smaller than the initial distance between the cold and warm walls of the bubble. This may be the reason why the stretching bubbles (Figure 2a), which usually had frost inside, were faster than the nonstretching bubbles, and is consistent with the observation that the bubble migration rates increased immediately after frosting [*Stehle*, 1967]. Once the bubble is completely filled with frost, the temperature gradient across the bubble becomes smaller and the migration rate decreases, as observed by *Stehle* [1967].

4. Implications for Snowball Earth

[24] The computed thickness of ice on the tropical ocean is sensitive to the bubble content of the near-surface ice. The blue-ice fields of the Trans-Antarctic mountains have an albedo of ~0.6 [*Warren and Brandt*, 2006], and for this albedo the model of *Pollard and Kasting* [2005] obtains kilometer-thick ice over the entire ocean. If bubble migration were to reduce the albedo to 0.47, Pollard and Kasting showed that the equilibrium ice thickness could be reduced to a few meters. If we assume the atmospheric pressure in the Neoproterozoic was similar to today's value, the upper limit for the migration rate of air bubbles in ice on a Snowball Ocean is the migration at 1 atm. Compared to brine migration [*Light et al.*, 2009], air bubble migration is slower for temperatures above approximately −30°C (Figure 7). In an ice cover over the ocean, air bubble migration would be faster close to the relatively warm ice-ocean interface and slower toward the colder air-ice interface. Figure 8 shows migration rates of brine and air bubbles computed as a function of depth in the ice for ice of total thickness 1–100 m, using a surface temperature of −20°C, which is considered to be the upper limit for the equatorial ocean in a global gla-

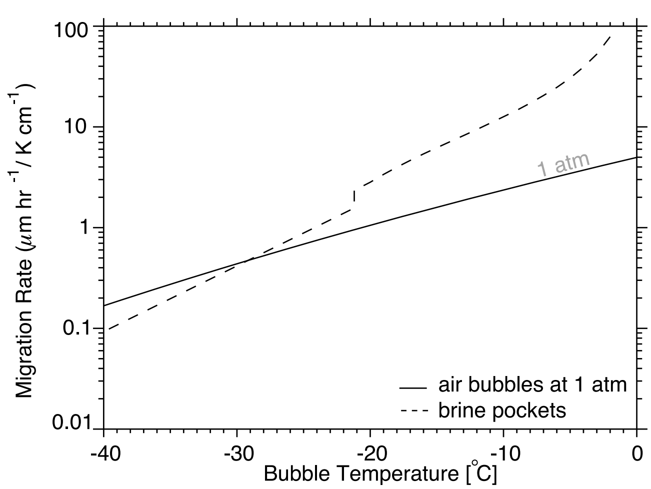

Figure 7. Theoretical migration rates [*Shreve*, 1967] of air bubbles at 1 atm (solid line) and measured migration rates of brine inclusions (dashed line) [*Light et al.*, 2009].

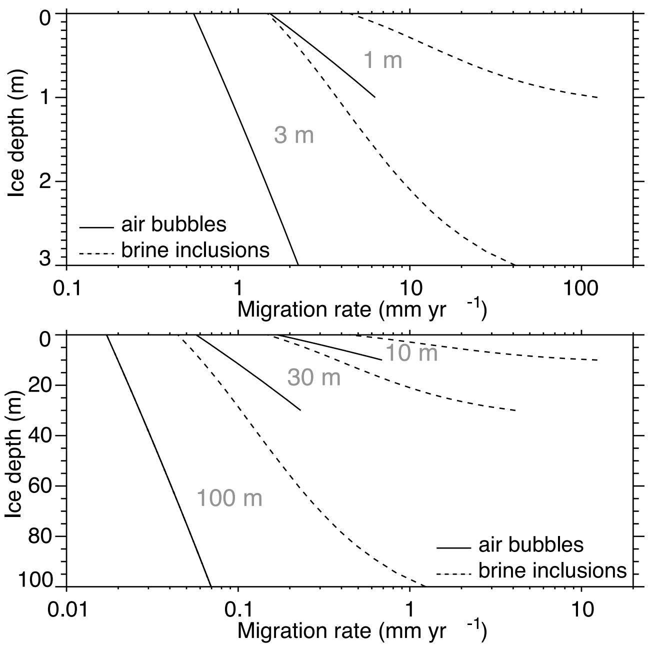

Figure 8. Migration rates of air bubbles and brine inclusions as functions of depth, in ice with various thicknesses. Temperature is increasing linearly from the top surface at $T_s = -20°C$ to the bottom surface at $T_f = -2°C$ (freezing temperature of sea ice). Solid line is for air bubble migration; dashed line is for brine migration, using temperature-dependent migration rates from Figure 7.

ciation [*Goodman and Pierrehumbert*, 2003]. We can compare these rates to expected sublimation rates, which were calculated to be 1–10 mm yr^{-1} during the coldest part of a snowball event [*Goodman and Pierrehumbert*, 2003; *Pollard and Kasting*, 2004; *Pierrehumbert*, 2005]. Even if the ice was as thin as 10 m, which is very thin for a "sea glacier," the migration rate of air bubbles would never exceed 1 mm yr^{-1} anywhere in the ice. For more realistic sea glacier thicknesses of hundreds of meters [*Goodman and Pierrehumbert*, 2003; *Goodman*, 2006], the migration rates are less than 0.1 mm yr^{-1}. It therefore seems unlikely that air bubble migration would outrun the advancing sublimation front and transform the glacial ice to a nearly bubble-free ice type, analogous to low-albedo marine ice [*Warren et al.*, 2002, Figure 8].

[25] There is a potential negative feedback relating bubble migration to climate. To illustrate this, the computations for Figure 8 are plotted in a different way in Figure 9: migration rate as a function of surface temperature T_s for several different specified ice thicknesses. Also plotted is the expected net sublimation rate for a frozen ocean near the equator, based on Figures 2 and 10 of *Pierrehumbert* [2005], which shows an equinoctial equatorial surface temperature of 238 K and net sublimation rate ($E - P$) of 3.5 mm yr^{-1} for 100 ppm of CO_2. We extrapolate this value of $E - P$ to higher and lower temperatures by assuming it is proportional to the equilibrium vapor pressure of ice [*Marti and Mauersberger*, 1993].

[26] For ice thicker than 0.3 m, the sublimation rate exceeds the migration rate at all temperatures. For a given temperature gradient ∇T, the migration is faster at higher temperature. But with higher T_s, ∇T is smaller because the temperature at the ice bottom is fixed at −2°C. Figure 9 shows that below −13°C the migration rate increases with temperature, but above −13°C it decreases with temperature because the reduction of ∇T dominates.

[27] Consider ice of fixed thickness 0.1 m with $T_s = -30°C$. The migration rate exceeds the sublimation rate, which would result in a reduction of albedo leading to surface warming, reaching a stable equilibrium temperature at $T_s \approx -20°C$. For higher temperatures the sublimation rate exceeds the migration rate, bringing the bubbles to the surface and maintaining a high albedo. We therefore conclude that bubble migration cannot contribute to deglaciation of a frozen ocean. Of course, for a surface temperature of −20°C or lower, in reality the ice will rapidly thicken (as it does on the modern polar oceans) so that the migration rate will drop to very low values, reinforcing our conclusion that the surface ice will retain a high bubble content.

[28] Our computations assumed a constant temperature gradient throughout the ice thickness. In nature, the diurnal and seasonal cycles of T_s will cause near-surface bubbles to move alternately up and down, instead of monotonically downward. During a summer afternoon, near-surface bubbles will migrate upward, thus aiding the effect of enhanced afternoon sublimation in maintaining the high albedo.

5. Conclusions

[29] To help assess the possible loss of bubbles from the surface of sea glaciers on Snowball Earth, a laboratory study was carried out to measure the migration rates of air bubbles in natural glacier ice as well as in laboratory-grown freshwater ice. Bubble migration rates (in both natural glacier ice and laboratory-grown samples) under a temperature gradient were observed to range from 0.03 μm h^{-1} (K cm^{-1})$^{-1}$ at −36°C to 5.45 μm h^{-1} (K cm^{-1})$^{-1}$ at −4°C. Most of the previous measurements [*Stehle*, 1967] of air bubble migration were made in laboratory-grown ice, and most of them only extend down to −14°C. Our observations agree with the earlier observations but also include bubbles at lower

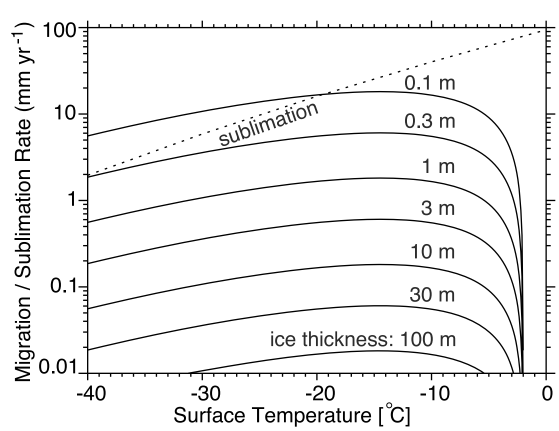

Figure 9. Migration rate of air bubbles as a function of surface temperature for different ice thicknesses (solid lines). The dashed line is the expected net tropical sublimation rate as a function of temperature, based on model results of *Pierrehumbert* [2005].

temperatures. Additionally, we assess the dependence of migration rate on bubble size, shape, and extent of frosting.

[30] Theoretical predictions of the migration rates for bubbles in freshwater ice were presented by *Shreve* [1967]. The theory indicates that bubble migration should depend on both temperature and internal bubble pressure. The present observations indicate a dependence on temperature that agrees with the theory. We also observed substantial variability of migration rates at given temperature and we believe this spread to reflect the variation in bubble pressure in the samples. We did not measure the pressure within individual bubbles, but believe that bubble pressures ranged from ~1 to ~6 atm based on the geometric depth of the ice sample from the glacial core and its proximity to the close-off depth.

[31] Our measurements indicate that bubble migration rates are too slow to outrun sublimation in the sea glaciers of Snowball Earth. If the ice surface did not regularly lose bubbles to downward migration, but was rather continually refreshed by constant surface ablation, we would expect the albedo of bare ice early in a snowball event to consistently remain high, despite the longevity of the ice. Later the albedo may be lowered by dust deposition [*Abbot and Pierrehumbert*, 2010].

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