

Refugium for surface life on Snowball Earth in a nearly-enclosed sea? A first simple model for sea-glacier invasion

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[1] The existence of photosynthetic eukaryotic algae during the so-called Snowball Earth events presents a conundrum. If thick ice covered the oceans, where could such life persist? Here we explore the possibility that photosynthetic life persisted at the end of long narrow seas, analogous to the modern-day Red Sea. In this first analytical model, we test the ability of the global sea glacier to penetrate a Red Sea analogue under climatic conditions appropriate during a Snowball Earth event. We find the Red Sea is long enough to provide a refugium only if certain ranges of climatic conditions are met. These ranges would likely expand if the restrictive effect of a narrow entrance strait is also considered. **Citation:** Campbell, A. J., E. D. Waddington, and S. G. Warren (2011), Refugium for surface life on Snowball Earth in a nearly-enclosed sea? A first simple model for sea-glacier invasion, *Geophys. Res. Lett.*, 38, L19502, doi:10.1029/2011GL048846.

1. Introduction

[2] During the low-latitude glaciation of the Neoproterozoic, 600–800 Ma [Evans, 2000; Harland, 1964], the upper ocean may have become almost completely covered with thick ice, commonly known as Snowball Earth events [Kirschvink, 1992; Hoffman *et al.*, 1998; Warren *et al.*, 2002; Pierrehumbert *et al.*, 2011]. Photosynthetic eukaryotic algae did survive these events [Knoll, 1992; Macdonald *et al.*, 2010], indicating that liquid water was maintained at some locations at or near the ocean surface. In the face of evidence for global ocean ice cover, how did open water and life persist? With an equilibrium ice thickness (set by geothermal flux) reaching hundreds of meters, but thicker at high latitude than at low latitude, the floating ice would flow equatorward as a “sea glacier” [Goodman and Pierrehumbert, 2003], threatening to invade any unprotected areas of open water. Small pools of open water could be maintained above geothermal hotspots on coastlines of volcanic islands [Hoffman and Schrag, 2000], where the water is shallow enough that the geothermal heat is not diffused laterally and that the sea glacier becomes grounded and slowed by friction with the sea floor.

[3] Here we investigate a possible class of refugia, isolated refuges for life, that could be much larger and also longer-lasting. A long narrow arm of a sea, for example as formed by continental rifting such as the modern Red Sea, located at low

latitude and surrounded by desert land, would be invaded by a tongue of the global sea glacier, but this tongue would be impeded by friction with the sidewalls and thinned by sublimation, so that its thickness would diminish to zero before reaching the end of the sea, if the sea is long enough. This possibility was mentioned by Warren *et al.* [2002] and considered likely by Pollard and Kasting [2005, 2006]. The low-albedo (non-glaciated) land surrounding the bay at the inland end of the narrow sea might keep the bay warm enough to allow open water, even while the rest of the ocean was kept cold by its high albedo. In a GCM simulation of Snowball Earth, a rectangular continent centered on the equator did have higher average temperatures than the equatorial ocean, particularly at the eastern end of the continent [Abbot and Pierrehumbert, 2010].

[4] In this paper we investigate only one aspect of this problem, namely the criterion for preventing the sea glacier from reaching the inland end of the sea. This is a necessary, but not sufficient, condition for the existence of a refugium. If the climate at the end of a narrow sea is too cold, the sea will fill with thick ice grown locally, even without any inflow from the global sea glacier. But even if the climate is warm enough to sustain open water or thin ice in the absence of ice flow, sea-glacier invasion could destroy the refugium. To isolate and explore this latter threat, we consider temperature regimes that are otherwise favorable to a refugium. In this first exercise, we further simplify the problem by considering only a sea of constant width W , which allows an analytical solution for the penetration distance L ; we therefore are not modeling the narrow strait that commonly constricts the entrance to nearly-enclosed seas on the modern Earth, such as the Bab al Mandeb, the Strait of Gibraltar, and the Bosphorus. In this model, the length-to-width ratio L/W at which the ice thickness diminishes to zero just at the far end of the narrow sea is a function of just three variables: the thickness H_0 at the entrance, the net sublimation rate at the top surface \dot{b} (positive where sublimation exceeds precipitation), and the atmospheric temperature T_s (which determines the ice temperature and effective ice softness and thus the flow velocity). We assume that there is no melting or freezing at the base of the sea glacier tongue. If the climate in the refugium is warm enough to support open water, there might be melting at the surface or base of the glacier, which would reduce L and make the probability of a refugium more likely than our computations here suggest.

2. Why Inland Seas Could Protect Refugia From Sea Glaciers

[5] The flow of a sea glacier into a channel would be resisted by drag stress along the sidewalls of the channel. If this channel is located in a region of net sublimation, the

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inflowing sea glacier will lose mass as it invades the channel. Alternatively, if the narrow sea is in a region of net accumulation, it would be surrounded by glaciated land and filled with snow-covered ice; thus an inland sea can provide a refugium only if it is in a region of net sublimation; i.e., surrounded by extensive desert. If the channel is sufficiently narrow, the resistive drag stresses impede the inflowing sea glacier, and sublimation causes the sea glacier to shrink to zero thickness after some length L .

[6] In order for an inland sea to provide a refugium for photosynthetic organisms during snowball events, several conditions must have been satisfied: (1) the inland sea must not be fully penetrated by a sea glacier; (2) the climate on the inland sea must be such as to maintain it either ice-free or covered by an ice layer sufficiently thin to allow photosynthesis below the ice; (3) the depth of the sea at its entrance, and throughout its length, must be great enough that seawater is able to flow under the sea glacier to replenish water loss from the refugium by evaporation/sublimation, and (4) water circulation in the inland sea must be adequate to allow nutrients to be delivered to organisms living in the bay at the landward end. Here, during this first exercise, we examine only the first condition.

3. Penetration Length of Sea Glaciers

[7] In order to determine whether inland seas could have remained free of sea-glacier ice, we estimate the penetration length L of a sea glacier flowing into an inland sea. The penetration length is defined as the distance a sea glacier, with an initial thickness H_0 , would have traveled before reaching zero thickness. To determine the penetration length L we derive an analytical solution for flow along a parallel-sided channel using force balance and continuity. *Weertman* [1957] determined the creep rate for an unconfined ice shelf with a uniform thickness and width. *Thomas* [1973] expanded on the work of *Weertman* [1957] by considering the creep rate of a confined ice shelf. By assuming a yield stress for side-wall shearing, he concluded that velocity in the downstream

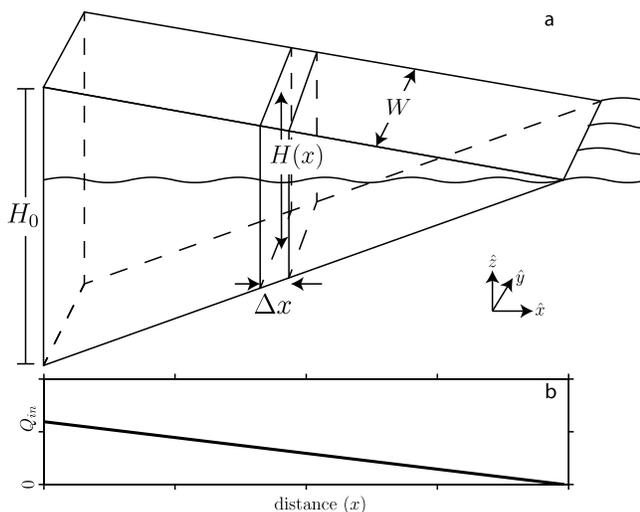


Figure 1. Cartoon schematic of a sea glacier penetrating a channel. (a) Ice thickness. (b) Ice flux. Both decrease linearly with distance, while mean across-channel velocity \bar{u} is uniform along the channel.

Table 1. Constants Used in Analysis

Name	Symbol	Value	Units
Temperature-independent ice softness parameter	A_0	4×10^{-13}	$\text{Pa}^{-3} \text{s}^{-1}$
Clausius-Clapeyron exponent	G	51	kJ/mol
acceleration of gravity	g	9.81	m/s^2
flow law exponent	n	3	dimensionless
activation energy for creep	Q_c	60	kJ/mol
ideal gas constant	R	8.314	$\text{J mol}^{-1} \text{K}^{-1}$
ice density	ρ_i	917	kg/m^3
seawater density	ρ_w	1043	kg/m^3

direction is constant. *Sanderson* [1979] found that velocity in the downstream direction did vary, but the variation was small. Therefore, we will assume that velocity is uniform in the downstream direction.

[8] Consider a sea glacier that enters a narrow channel with a uniform width W and vertical walls (Figure 1a). We choose the x -direction to be horizontal and parallel to the channel walls, the y -direction to be perpendicular to the channel walls, and the z -direction to be vertical, positive upward. The centerline of the sea glacier is at $y = 0$.

[9] Environmental controls such as temperature and net rate of sublimation also control the penetration length of a sea glacier. In Snowball Earth models, zonally-averaged mean annual surface temperatures ranged from -50°C to -20°C at latitudes where net sublimation was likely [*Goodman*, 2006; *Pierrehumbert et al.*, 2011]. Ice is softer at warmer temperatures, hence a warmer sea glacier will flow faster so may penetrate farther. Basal temperatures were calculated by adjusting the melting point of ice for pressure and salt content. Salt content was estimated by assuming present-day total salinity and total volume of the ocean, and adjusting the salinity taking into account the salt-free ice covering the ocean during a Snowball Earth event. A relationship between ice temperature and the softness parameter $A(T)$ has been derived empirically [*Cuffey and Paterson*, 2010, p. 75] and has a functional form

$$A(T) = A_0 \exp\left(\frac{-Q_c}{RT}\right) \quad (1)$$

where A_0 represents a temperature-independent softness coefficient, Q_c represents the activation energy for creep, R represents the ideal gas constant, and T represents ice temperature. Values for the constants are given in Table 1. Because the rate of vertical advection of heat is small compared to the heat conduction rate, we assume a linear temperature profile throughout the ice thickness. Then we calculate an effective isothermal ice softness parameter A that satisfies force balance and produces the same ice flux in the channel as the value obtained by using the nonuniform temperature profile; a full explanation is given in the auxiliary material.¹ This new isothermal ice softness parameter A is used in place of the depth-dependent A in all equations that follow. Using an effective isothermal ice temperature to determine ice softness allows us to solve the problem using simpler isothermal equations.

[10] The mass loss has two components, sublimation at the surface of the sea glacier, and melting at its base where

¹Auxiliary materials are available in the HTML. doi:10.1029/2011GL048846.

it contacts ocean water. The net rate of sublimation, \dot{b} , is poorly constrained. In *Pierrehumbert's* [2005] model [see also *Pierrehumbert et al.*, 2011, Figure 7], the zonal average net sublimation rate peaked at the equator with a value of 5 mm/year, for 2000 ppm CO₂. During the coldest phase of a Snowball Earth event, CO₂ may have been just 100 ppm, and corresponding net sublimation only 3 mm/year [Pierrehumbert, 2005, Figure 10]. *Goodman* [2006] calculated that mass loss through basal melting is insignificant compared to sublimation losses at equatorial latitudes. Therefore we use a parameter space to calculate penetration lengths where net rate of sublimation and ice softness are varied, while other parameters are held constant (Table 1).

3.1. Channel Velocity

[11] *Nye* [1965] solved for the velocity for a glacier with infinite depth and uniform width W where flow is resisted by two parallel sidewalls. *Nye's* equation 7 neglected vertical shearing because of the infinite thickness of the glacier. A sea glacier entering a narrow channel does not undergo vertical shearing because the sea glacier is floating and water cannot support a shear stress, so *Nye's* solution is appropriate:

$$u(x, y) = \frac{W}{2} \frac{A(x)k(x)^n}{n+1} \left(1 - \left| \frac{2y}{W} \right|^{n+1} \right) \quad (2)$$

where u represents velocity in the x -direction, k represents resistive drag stress at the walls, A represents ice softness, and n represents the exponent in the *Glen* ice flow law [Glen, 1955]. *Nye's* solution assumes no variation of velocity in the x -direction, but variation of u in the x -direction is permitted if k varies as a function of x . Variation of k in the z -direction is not permitted, because there is no basal shear stress to cause a vertical gradient of velocity. Equation (2) can be integrated across the channel to calculate a mean velocity, $\bar{u}(x)$

$$\bar{u}(x) = \frac{W}{2} \frac{A(x)k(x)^n}{n+2} \quad (3)$$

3.2. Continuity

[12] We use the time-independent mass conservation equation

$$\nabla \cdot (\bar{\mathbf{u}}H) + \dot{b} = 0, \quad (4)$$

where $\bar{\mathbf{u}}$ represents the ice-velocity vector, H represents local ice thickness, and \dot{b} represents the net sublimation rate at the upper surface (\dot{b} is positive for loss in volume). Motion is entirely in the x -direction, so equation (4) can be written as

$$\frac{d\bar{u}}{dx} H(x) + \bar{u}(x) \frac{dH}{dx} = -\dot{b}(x) \quad (5)$$

Substituting equation (3) into equation (5) results in

$$H(x) \frac{W}{2} \frac{A(x)}{n+2} \frac{d}{dx} (k(x)^n) + \frac{W}{2} \frac{A(x)k(x)^n}{n+2} \frac{dH}{dx} = -\dot{b}(x) \quad (6)$$

3.3. Force Balance

[13] To solve equation (6), we must determine the resistive drag stress k as a function of x . We use a force-balance approach where we assume that the pressure gradient forces in a sea glacier flowing along a narrow channel are entirely balanced by the resistive drag force at the sides; a full explanation is given in the auxiliary material, where equation (S12) shows that the resistive drag stress at the walls is

$$k(x) = -\frac{W}{2} \Gamma \frac{dH}{dx} \quad (7)$$

where Γ is defined as

$$\Gamma \equiv \rho_i g \left(1 - \frac{\rho_i}{\rho_w} \right) \quad (8)$$

and ρ_i represents the density of ice, ρ_w represents the density of sea water, and g represents the acceleration due to gravity. By substituting equation (7) into equation (6), we can write a differential equation in terms of H

$$\frac{W}{2} \frac{A(x)}{n+2} \left(\frac{-W}{2} \Gamma \right)^n \left[nH(x) \left(\frac{dH}{dx} \right)^{n-1} \frac{d^2H}{dx^2} + \left(\frac{dH}{dx} \right)^{n+1} \right] = -\dot{b}(x) \quad (9)$$

Equation (9) with the boundary condition $H(0) = H_0$ has a solution in which H varies linearly with x :

$$H(x) = H_0 \left(1 - \frac{x}{L} \right) \quad (10)$$

$$\frac{dH}{dx} = -\frac{H_0}{L} \quad (11)$$

$$\frac{d^2H}{dx^2} = 0 \quad (12)$$

The ratio L/W can be found by substituting equations (10), (11), and (12) into equation (6) to get

$$\frac{L}{W} = \frac{H_0}{D} \quad (13)$$

where

$$D = 2 \left(\frac{\dot{b}(n+2)}{A\Gamma^n} \right)^{\frac{1}{n+1}} \quad (14)$$

is a characteristic length that depends on temperature, through A , and net sublimation rate \dot{b} .

[14] Mean velocity, \bar{u} , across the channel can be calculated as

$$\bar{u} = \frac{W}{2} \left(\frac{A(\Gamma\dot{b})^n}{n+2} \right)^{\frac{1}{n+1}} \quad (15)$$

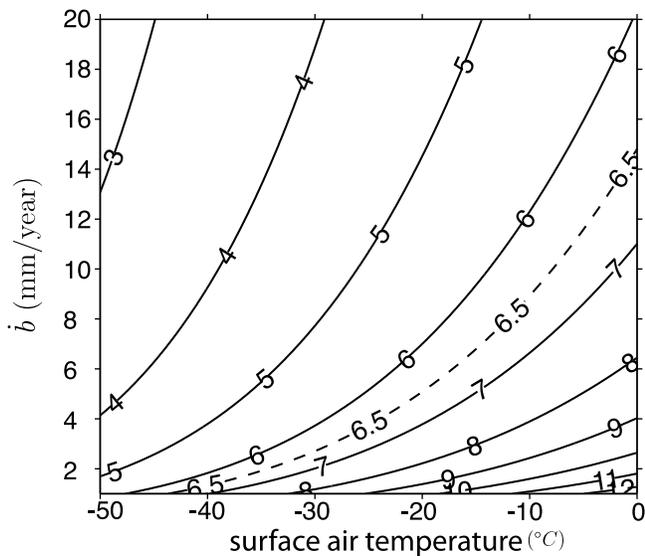


Figure 2. Solid contours represent penetration length-to-width ratio L/W as a function of net sublimation rate \dot{b} and surface ice temperature T_s for a sea glacier with an initial thickness $H_0 = 650$ m, entering a narrow channel. Dashed line represents the 6.5 L/W ratio for the Red Sea. Atmospheric conditions to the left of the dashed line allow a refugium to avoid being over-ridden at the end of the Red Sea analogue.

and flux, Q_0 , entering the channel through the upstream end from the global sea glacier can be calculated as

$$Q_0 = WH_0\bar{u}. \quad (16)$$

4. Application to a Modern Analogue

[15] Penetration length scales linearly with channel width (equation (13)); therefore, penetration length L can be non-dimensionalized in terms of channel width W . Figure 2 demonstrates that a sea glacier can penetrate farther into a narrow channel if it has lower net sublimation rate \dot{b} , and/or warmer air temperatures represented by larger ice softness parameter A .

[16] As a characteristic inland sea we use the Red Sea, a nearly enclosed basin in the Red Sea Rift, a spreading center between the African and Arabian plates. Because the rifting of continental plates is caused by convection in the Earth's mantle, which was presumably not significantly affected by Snowball Earth events, it is reasonable to assume that inland seas like the Red Sea existed during Snowball Earth events.

[17] To determine the penetration length of a sea glacier moving into a Red Sea analogue, we approximate the sea by a rectangle 200 km wide and 1300 km long; the length-to-width ratio L/W is 6.5. We assume that the upstream thickness H_0 of a sea glacier at the mouth of a Red Sea analogue is 650 m, consistent with ice thickness of the global sea glacier in regions of net sublimation [Goodman, 2006]; however, penetration length scales linearly with H_0 (equation (13)) so a thicker global sea glacier could penetrate proportionally farther. The length of the Red Sea exceeds the penetration length of the sea glacier if the combination of ice temperature and sublimation rate falls to the left of the

dashed line in Figure 2. The along-channel ice thickness $H(x)$, and ice flux $Q(x)$ for a sea glacier, decrease linearly with distance Figure 1 while mean along-channel velocity \bar{u} remains constant.

[18] The planetary climate of a Snowball Earth event warms over time through either the build-up of atmospheric CO_2 or changes in planetary albedo [Abbot and Pierrehumbert, 2010]. As the planet warms, the length-to-width ratio L/W of a sea glacier invading a narrow channel may change. We explore changes in L/W by exploring changes in the local net sublimation rate \dot{b} and ice softness A due to a change in atmospheric temperature T_s . Here, we assume that changes \dot{b} are proportional to changes in the saturation vapor pressure from a Clausius-Clapeyron relation

$$\dot{b}(T_s) = c \exp\left(\frac{-G}{T_s R}\right) \quad (17)$$

where c represents a constant based on initial conditions, and G represents the Clausius-Clapeyron exponent from Marti and Mauersberger [1993]. Atmospheric temperature T_s changes slowly enough to allow the sea glacier to always be in an equilibrium steady-state because the time scale of thermal diffusion in the penetrating sea glacier is much less than the time scale for global changes in T_s . Therefore T_s can be used as a proxy for time. Figure 3a shows an example calculation with initial climate conditions of $T_s = -50^\circ\text{C}$ and $\dot{b} = 1$ mm/year. The length-to-width ratio L/W of the sea glacier, described by equation (13), decreases in a warming world because increases in length due to increased ice softness are smaller than decreases in length due to increased net sublimation (Figure 3b). Both sublimation and shearing

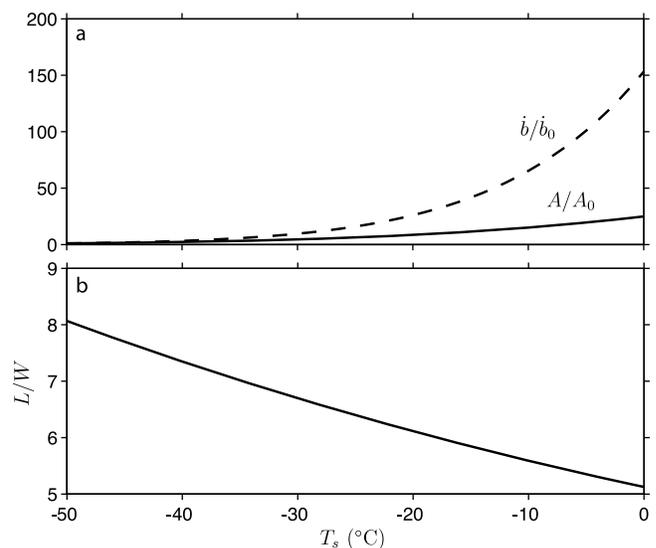


Figure 3. (a) How increasing atmospheric temperatures during deglaciation affect net sublimation rate \dot{b} , relative to an initial value \dot{b}_0 of 1 mm/year (dashed line), and ice softness A , relative to an initial value A_0 set by a surface temperature $T_s = -50^\circ\text{C}$ (solid line). Net sublimation rate \dot{b} is more sensitive than ice softness A to rising surface temperature T_s . (b) Resulting length-to-width ratio L/W of an invading sea glacier, calculated from equation (13). For this particular example, the initial upstream ice thickness is $H_0 = 650$ m.

require breaking of hydrogen bonds. Correspondingly, the Clausius-Clapeyron exponent (51 kJ/mol) is similar to the activation energy for creep (60 kJ/mol); each is approximately the energy required to break two hydrogen bonds. The reason that A increases more slowly than b in Figure 3b is that b is determined by the surface temperature whereas A is determined by an effective depth-averaged temperature including the influence of warm ice at the bottom (-2°C) for all values of the surface temperature. Although this is a specific example, other reasonable initial conditions produce qualitatively similar results. In addition, this approach does not take into account surface and basal melting processes that would be expected once atmospheric conditions are near the melting temperature, and furthermore the upstream thickness H_0 of the global sea glacier entering the channel would decrease in a warming world. Both of these effects would decrease L/W for an invading sea glacier and L/W would be less than its initial value throughout the deglaciation.

5. Conclusions

[19] As modeled here, an analogue to the Red Sea is long enough to have provided a refugium for photosynthetic organisms during Snowball Earth events, only if certain surface temperature and net sublimation conditions were met. The penetration length of an invading sea glacier continuously decreases as the planet warms during a Snowball Earth event. In future work, the restraining effect of a narrow entrance strait to the sea will be considered. That investigation will require a numerical model, but will likely allow a refugium to exist over a wider range of atmospheric temperatures and net sublimation rates.

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