Refugium for surface life on Snowball Earth in a nearly enclosed sea? A numerical solution for sea-glacier invasion through a narrow strait

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Abstract

Where photosynthetic eukaryotic organisms survived during the Snowball Earth events of the Neoproterozoic remains unclear. Our previous research tested whether a narrow arm of the ocean, similar to the modern Red Sea, could have been a refugium for photosynthetic eukaryotes during the Snowball Earth. Using an analytical ice-flow model, we demonstrated that a limited range of climate conditions could restrict sea-glacier flow sufficiently to allow an arm of the sea to remain partially free from sea-glacier penetration, a necessary condition for it to act as a refugium. Here we expand on the previous study, using a numerical ice-flow model, with the ability to capture additional physics, to calculate sea-glacier penetration, and to explore the effect of a channel with a narrow entrance. The climatic conditions are made self-consistent by linking sublimation rate to surface temperature. As expected, the narrow entrance allows parts of the nearly enclosed sea to remain safe from sea-glacier penetration for a wider range of climate conditions.

1. Introduction

During the "Snowball Earth" events of the Neoproterozoic, approximately 600–800 Ma [Hoffman and Schrag, 2002], the entire upper ocean surface may have been covered with "sea glaciers," ice several hundreds of meters thick that are thick enough to deform under their own weight [Goodman and Pierrehumbert, 2003; Goodman, 2006; Li and Pierrehumbert, 2011]. Photosynthetic eukaryotic algae existed immediately prior to and after these events [Knoll, 1992; Macdonald et al., 2010] so they are presumed to have survived during Snowball Earth. These algae require both liquid water and sunlight. Our previous research [Campbell et al., 2011] explored the ability of a nearly enclosed sea, connected to the ocean by a narrow channel, to provide a refugium for photosynthetic eukaryotic life during a Snowball Earth event. (Figure 1 provides an illustration of how we envision a sea-glacier penetrating into an inland sea.) We demonstrated, using an analytical solution of momentum balance and continuity equations, that the landward end of the sea could remain free from sea-glacier penetration, provided particular geometric and climatic conditions existed. If an inland sea is not fully penetrated by a sea glacier, then it has achieved one of the conditions needed to provide a refugium for photosynthetic eukaryotic life. In addition, the sea would have to be located in a desert region, where evaporation/sublimation exceeds precipitation, and the refugium's temperature would have to be warm enough that thick sea ice did not grow locally. The flow of ice through constrictions was also studied by Tziperman et al. [2012].

Section 2 reviews the previously developed model of sea-glacier penetration of Campbell et al. [2011]. Section 3 describes the newly developed numerical model of sea-glacier penetration. That section includes descriptions of the model's body equations, boundary conditions, and parameters, as well as modifications made to the model to ensure numerical stability, and our solution scheme. Section 4 describes the two experiments performed using the numerical model: a comparison experiment where we compare the new model results to those of Campbell et al. [2011] and an experiment where we vary the entrance width of the channel. Section 5 describes an experiment in which we couple a thermal evolution model to the sea-glacier flow model to demonstrate that sea-glacier penetration is reduced in a channel that is warmer at its far end than at its entrance, in accord with the climate necessary to prevent sea-ice growth locally at the far end. In section 6, we discuss the results and assumptions of the experiments, including the discovery of upstream sea-glacier-free zones, the atmospheric constraints that would be required for incomplete
sea-glacier penetration, and our choice to not address the effect of submarine melting. Section 7 summarizes our conclusions that narrow entrances widen the range of climate conditions that allow for inland seas to act as refugia for photosynthetic eukaryotic algae during Snowball Earth events.

2. Penetration of Sea Glaciers

Campbell et al. [2011] used an analytical solution of momentum balance and continuity equations to calculate the distance that an incoming sea glacier would advance up the channel, referred to as penetration length. That analytical model assumed a uniform channel width. The mean-annual surface temperature and sublimation rate were independently specified, both were spatially uniform and constant in time. That model included only lateral shear stresses and assumed no longitudinal stretching; those conditions ensured a linear thickness gradient in the downstream direction and no thickness gradient in the lateral direction. Penetration length was determined to be a function of upstream ice thickness, channel width, ice softness (which is controlled by the mean-annual surface temperature), and sublimation rate. Campbell et al. [2011] did not address the effect of melting at the lower surface of the sea glacier. When the model was applied to a channel with the same mean length and mean width as the Red Sea (a modern analogue, because continental rifting, which is responsible for the Red Sea, would have been active during the Neoproterozoic), a restrictive range of surface temperatures and net sublimation rates allow the Red Sea to remain free of sea-glacier penetration. Campbell et al. [2011] concluded with a suggestion that a narrow entrance to the channel, a feature observed in the modern Red Sea, would decrease the penetration length of an incoming sea glacier by restricting the flow of ice into the channel, expanding the possible range of conditions that would allow the channel to be partially free from sea-glacier penetration. Campbell et al. [2011] contained an error in its calculations of sea-glacier penetration, which is corrected in Appendix A of this paper.

3. Model

Previously, we developed an analytical model to calculate sea-glacier penetration length $L$ into a narrow channel [Campbell et al., 2011] with uniform width. That solution is limited in applicability because it calculates only the lateral shearing stress component and assumes a uniform lateral ice thickness and a linear downstream thickness profile. That model is not applicable to a channel with a varying width.

Here we develop a numerical model that resolves both lateral shearing and longitudinal stretching. This new model allows for a freely developing thickness field and is able to simulate the effect of a narrow entrance. In this new model, we couple equations of motion based on the Shallow Shelf Approximation to a time-independent mass conservation equation. We specify mean-annual surface temperature and sublimation rate. Then a Glen’s Flow Law viscosity is used with an ice softness parameter based on the mean-annual surface temperature. To determine penetration length $L$, we iteratively the change channel length until the dynamic ice flux entering the channel balances the kinematic ice flux lost by sublimation. These equations are solved using a commercially available finite element solver, COMSOL Multiphysics (comsol.com).

3.1. Body Equations

We use equations of motion that describe the flow of a layer of floating ice with thickness $h$ much less than horizontal extent, commonly called the Shallow Shelf Approximation (SSA) [Morland, 1987; MacAyeal et al., 1996]
where \( x \) and \( y \) represent the horizontal spatial coordinates, \( u \) and \( v \) represent the corresponding velocities, \( v \) represents the viscosity, \( h \) represents the ice thickness, \( \rho_i \) represents the ice density, \( g \) represents the acceleration of gravity, and \( S \) represents the surface elevation above sea level. The SSA assumes the ice is in hydrostatic equilibrium (i.e., floating); therefore, surface elevation is related to local thickness by a buoyancy relation

\[
S = \left(1 - \frac{\rho_i}{\rho_w}\right) h, \tag{3}
\]

where \( \rho_w \) represents the water density.

Coupled to the SSA is a time-independent mass conservation equation

\[
\nabla \cdot (\overline{u} h) + \dot{b} = 0, \tag{4}
\]

where \( \dot{b} \) represents the net sublimation rate and \( \overline{u} \) represents the velocity vector.

An effective viscosity is used that embodies Glen’s Flow Law [Glen, 1955]

\[
v = A^{-1/n} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \frac{1}{4} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \frac{1}{4} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 \right]^{\frac{1}{2}}, \tag{5}
\]

where \( n \) represents a flow rate exponent and \( A \) represents a temperature dependent softness parameter (described below in section 3.4).

### 3.2. Boundary Conditions

To solve the equations of motion and the mass conservation equation, boundary conditions must be imposed. There are three types of boundaries considered in this 2-D model: the entrance, the sidewalls, and the sea-glacier front. At the entrance, ice moves into the channel from the global sea glacier. To represent the entrance boundary condition, a uniform thickness \( H_0 \) is maintained along the entrance boundary. Ice is frozen stuck to the sidewalls. To represent the sidewalls, a no-slip boundary condition \( \left( \overline{u} = 0 \right) \) is imposed.

At the sea-glacier front, ice contacts seawater where it is in hydrostatic equilibrium

\[
\int_{-h}^{1-\frac{\rho_i}{\rho_w}} T \cdot \overline{n} \, dz = -\frac{\rho_w g}{2} \left( \frac{\rho_i}{\rho_w} \right)^2 \overline{n}, \tag{6}
\]

where \( \overline{T} \) represents the stress tensor, and \( \overline{n} \) represents an outward pointing normal vector.

### 3.3. Modifications

To solve the SSA, ice thickness \( h \) must be positive throughout the entire model domain, so we specify a small minimum thickness \( h_{\text{min}} \). To enforce this condition, the SSA, sea-glacier front boundary condition, and the continuity condition are modified. If a particular point in the model has a thickness less than \( h_{\text{min}} \), the SSA and sea-glacier front boundary condition use \( h_{\text{min}} \) in place of \( h \). The continuity equation is modified so that if a particular node has thickness less than \( h_{\text{min}} \) it will adjust the thickness to be \( h_{\text{min}} \) on the next iteration. The penetration length shows very little dependence on the choice of \( h_{\text{min}} \) in the range from 5 to 50 m.

The continuity equation is further modified to allow for artificial diffusion to avoid numerical instability. A set of representative experiments shows that neither of these modifications significantly affects the final solution. In these experiments, we first achieve a stable sea-glacier model using the above method; then a
3.4. Model Parameters

Values of the parameters used in the model are summarized in Table 1. Upstream ice thickness $H_0$ is chosen to be 650 m, based on equatorial ice-thickness model results from Goodman [2006]. Channel width $W$ of 200 km is chosen to be similar to the modern Red Sea. Basal temperatures were calculated by adjusting the melting point of ice for pressure and salt content. Salt content was estimated by assuming present-day total salinity and total volume of the ocean, and adjusting the salinity taking into account the salt-free ice covering the ocean during a Snowball Earth event; this gives a salinity about 20% greater than modern, and a freezing temperature of $-2.3^\circ$C, instead of $-1.9^\circ$C. A relationship between the ice temperature and the softness parameter $A(T)$ has been derived empirically [Cuffey and Paterson, 2010, p. 75] and has a functional form

$$A(T) = A_0 \exp \left( \frac{-Q_c}{RT} \right), \quad (7)$$

where $A_0$ represents a temperature-independent ice softness coefficient, $Q_c$ represents the activation energy for creep, $R$ represents the ideal gas constant, and $T$ represents the ice temperature in Kelvin. Because the rate of vertical advection of heat is small compared to the heat conduction rate, we assume a linear temperature profile throughout the ice thickness. Then we calculate an effective isothermal ice softness parameter $A$ that satisfies force balance and produces the same ice flux in the channel as the value obtained by using the nonuniform temperature profile; a full explanation is given in Campbell et al. [2011, supporting information]. This new isothermal ice softness parameter $A$ is used in place of the depth-dependent $A$ in all equations that follow. This approximation allows us to use the SSA.

3.5. Solution Scheme

A rectangular model domain is used to represent the inland sea, with a channel width $W$ and an adjustable channel length $L$. The geometry uses a mesh with triangular elements. The model is iterated upon until consistent fields of velocity, thickness, and viscosity are obtained. A dynamic flux is calculated by integrating ice flow across the entrance. A kinematic flux is calculated by integrating sublimation over the upper surface of the sea glacier. Channel length is adjusted by a root-finding scheme until the dynamic flux balances the kinematic flux, at which point we declare the channel length to be the penetration length of the sea glacier.

4. Experiments

We report on two sets of experiments: a validation experiment and a test of the effect of a narrow entrance.

4.1. Comparison to Analytical Model

We apply the numerical model to the idealized geometry used in the analytical model [Campbell et al., 2011], for the same range of surface temperatures and sublimation rates (Figure 2). Penetration lengths

Table 1. Constants Used in Analysis

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature-independent ice softness coefficient</td>
<td>$A_0$</td>
<td>$4 \times 10^{-13}$</td>
<td>Pa$^{-3}$s$^{-1}$</td>
</tr>
<tr>
<td>Upstream ice thickness</td>
<td>$H_0$</td>
<td>650 m</td>
<td></td>
</tr>
<tr>
<td>Minimum ice thickness</td>
<td>$h_{\text{min}}$</td>
<td>20 m</td>
<td></td>
</tr>
<tr>
<td>Clausius-Clapeyron exponent</td>
<td>$g$</td>
<td>51 kJ/mol</td>
<td></td>
</tr>
<tr>
<td>Acceleration of gravity</td>
<td>$g$</td>
<td>9.81 m/s$^2$</td>
<td></td>
</tr>
<tr>
<td>Flow law exponent</td>
<td>$n$</td>
<td>3 Dimensionless</td>
<td></td>
</tr>
<tr>
<td>Activation energy for creep</td>
<td>$Q_c$</td>
<td>60 kJ/mol</td>
<td></td>
</tr>
<tr>
<td>Ideal gas constant</td>
<td>$R$</td>
<td>8.314 J mol$^{-1}$K$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Main channel width</td>
<td>$W$</td>
<td>200 km</td>
<td></td>
</tr>
<tr>
<td>Thermal diffusivity of ice</td>
<td>$\kappa$</td>
<td>$1.2 \times 10^{-6}$</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>Ice density</td>
<td>$\rho_i$</td>
<td>917 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Seawater density</td>
<td>$\rho_w$</td>
<td>1043 kg/m$^3$</td>
<td></td>
</tr>
</tbody>
</table>

new model without these modifications is initialized with the solution from our stable solution. These models do achieve stability, without significant changes to the velocity and thickness fields.
calculated by the numerical model are larger than those of the analytical solution, by up to a factor of 2, particularly at low net sublimation rates. The effect is diminished at larger sublimation rates, with at most a 30% increase in penetration length at net sublimation rates above 10 mm/yr.

4.2. Narrow Entrance Experiment

To determine the effect of a narrow entrance, we allow the width of the entrance to change while the width of the main channel remains constant. For this experiment, the boundaries adjacent to the entrance are treated as sidewalls. We calculate penetration length over the same range of mean-annual surface temperature and sublimation rate as in Figure 2 and in addition we vary the ratio of entrance width to channel width, $W_s/W$, from 5% to 100%. An example calculation of penetration length to channel width ratio $L/W$ is shown in Figure 3a for $W_s/W$ of 30%. In Campbell et al., [2011], we used the Red Sea as an analogue for the type of inland seas we expect to have existed during Snowball Earth events. The $L/W$ of the Red Sea is 6.5, but other inland seas have differing $L/W$. The choice of $L/W = 6.5$ as a cutoff is simply a baseline for comparison. A refugium is possible only for combinations of $T_s$ and $b$ above the dashed line. Figure 3b tracks the 6.5 $L/W$ contour as a function of $W_s/W$, showing that the range of climates permitting a refugium expands
as the entrance narrows. Figure 4a shows the thickness and velocity fields for a sea glacier entering a channel where $T_s = 52^\circ C$, $b = 18$ mm /yr, and $W_s/W = 0.125$.

5. Thermal Evolution of an Invading Sea Glacier

In order for an inland sea to act as a refugium the atmospheric temperature must be warm enough that thick sea ice will not grow locally. The far end of the inland sea must therefore not be much below the melting temperature of ice. This situation is plausible because the oasis at the end of the channel would be surrounded by unglaciated desert land with albedo much lower than that of the ice-covered ocean.

As a cold sea glacier begins to invade into a warm inland sea, the sea glacier will warm, softening the ice. This effect will accelerate the sea glacier causing it to penetrate farther into the inland sea. However, as an inland sea goes from a cold temperature near its entrance, toward a warmer region, the associated net sublimation rate will increase. This effect reduces the penetration length of the sea glacier into the inland sea. The effects of increasing ice softness and increasing net sublimation are therefore in competition in determining sea-glacier penetration. In this section, we couple a thermal evolution model to the ice-flow model to determine whether the combination of these effects reduces or increases sea-glacier penetration.

5.1. Dependence of Net Sublimation Rate on Temperature

Atmospheric temperature varies over the inland sea; the ice surface temperature, and the net sublimation rate will vary correspondingly. Here we prescribe the surface temperature field and calculate the associated net sublimation rate. At the upstream end of the inland sea we choose an initial surface temperature $T_s$ to
be the equatorial surface temperature of the global sea glacier, with corresponding sublimation rate $b_0$. We linearly vary the atmospheric temperature along the downstream direction of the sea glacier (the $x$ direction, with $x = 0$ at the entrance), assuming the temperature is uniform across the width of the channel (the $y$ direction),

$$T_s(x, y) = T_{s0} + \frac{(T_m - T_{s0})x}{L_t}, \quad (8)$$

where $L_t$ represents the transition distance for the inland sea to reach $T_m = 273$ K. Here we assume that changes in $b$ are proportional to changes in the saturation vapor pressure according to the Clausius-Clapeyron relation

$$b(T_s) = c \exp\left(-\frac{G}{RT_s}\right), \quad (9)$$

where $c$ represents a constant which gives $b(T_{s0}) = b_0$, and $G$ represents the Clausius-Clapeyron exponent from Marti and Mauersberger [1993].

### 5.2. Thermal Evolution Model

To calculate the thermal evolution of a penetrating sea glacier, we couple a 1-D thermal diffusion and vertical advection model following Hooke [2005]

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z}, \quad (10)$$

where $t$ represents the time, $\kappa$ represents the thermal diffusivity of ice, $z$ represents a positive-upward vertical coordinate, and $w$ represents the corresponding vertical velocity. Vertical velocity is assumed to go linearly to zero at the base of the sea glacier:

$$w = \frac{z}{h} w_s, \quad (11)$$

where $w_s$ represents the vertical velocity at the surface of the glacier. For this calculation, $z$ is set to be zero at the local sea-glacier base and vertical velocity at the surface $w_s$ is calculated to balance the sublimation rate and the depth-averaged strain rate.
The model is forced by temperature boundary conditions of $T_s$ at the upper surface and the pressure-dependent melting temperature (see section 3.4) at the basal surface. We choose an upper-surface temperature $T_{s0}$ and net sublimation rate $b_0$ at the entrance based on the ranges provided in Figure 2.

5.2.1. Model Coupling
The thermal evolution model generates a centerline temperature profile. For every $x$ position, we calculate an integrated ice softness $A$ (see section 3.4) and we assume it is laterally ($y$ direction) uniform. We then use these ice softnesses and their associated net sublimation rates in the ice-flow model described in section 3. With a new centerline velocity, we iteratively feed results back and forth between the thermal model and the ice-flow model until a stable solution is reached.

5.3. Results of Thermal Model
Figure 5a shows the temperature profile for an invading sea glacier with $T_{s0} = -30^\circ C$, $b_0 = 10.5 \text{ mm/yr}$, and $W_s/W = 0.3$. The values of $T_{s0}$ and $b_0$ are consistent with climatic conditions expected for the equator of Snowball Earth [Pierrehumbert et al., 2011]. Figure 5c shows that in a warming channel, the ice softness $A$ increases more slowly than the net sublimation rate $b$. Thus, the penetration length of the sea glacier is reduced in comparison to a channel with uniform surface temperature and sublimation rate. Although this is a specific example, the pattern of diminished sea-glacier penetration is true for all combinations of $T_s$, $b$, and $W$ explored in this work.

6. Discussion
6.1. Sea-Glacier-Free Zones
In order to solve the SSA, a small, positive minimum ice thickness $h_{\text{min}}$ must be enforced throughout the model domain. The minimum ice thickness chosen here is of the same order of magnitude as the greatest ice thickness that allows for photosynthesis to occur underneath bubble-free ice [Warren et al., 2002]. Therefore, sea-glacier-free zones may have acted as refugia, provided that thick sea ice did not grow locally.

In all experiments, ice thickness diminished to $h_{\text{min}}$ at the downstream end after traveling a sufficient distance. The narrow-entrance experiment also revealed areas adjacent to the entrance where ice thickness reached $h_{\text{min}}$ (Figure 4). Ice enters the channel through a narrow entrance and is forced to flow laterally and, at some locations, backward to flow into areas adjacent to the entrance. In the example shown in Figure 4, there are areas 20–40 km away from the channel entrance where $h = h_{\text{min}}$. In the model, there are “sea-glacier-free” regions because we artificially prevented $h < h_{\text{min}}$, so we interpret these areas to be completely free of sea-glacier ice. We had anticipated that sea-glacier-free areas could exist only far downstream of the channel entrance; however, for a channel with a narrow entrance, it is possible to have ice-free areas located near the entrance on the upstream end of the channel. Not all experiments produced sea-glacier-free zones near the entrance; they were found primarily in experiments with narrow entrances, high net
sublimation, and low surface temperature. In fact, the net sublimation rate specified for Figure 4 (10.5 mm/yr) is probably larger than would occur for the specified $T_s = -50^\circ C$.

Sea-glacier-free zones near the channel entrance would grow thick ice locally if they are located in a cold region of the inland sea. We conclude that none of the sea-glacier-free zones observed in our models near the channel entrance would act as refugia, because they are only observed with colder temperatures that would locally generate thick sea-ice.

The discovery that sea-glacier-free zones can be located away from the downstream end of a channel indicates that our assumption of a rectangular channel might limit our ability to discover and categorize other ice-free areas that are possible for ice entering a narrow channel. Natural channels are not perfectly rectangular. It is possible that shallow spots and side channels could generate ice shadows and ice-free areas that are not located at the downstream end of a single main channel. Obstructions upstream from the terminus, but where atmospheric temperatures are warm enough, may generate sea-glacier-free zones that could act as refugia.

### 6.2. Submarine Melting

Submarine melting is not explicitly taken into account in this study. However, the presence of submarine melting or freezing could be accounted for in this model by adjusting the net sublimation rate $b$ in the continuity equation (equation (4)). Ashkenazy et al. [2013] have used a coupled ocean-atmosphere GCM to calculate both surface net sublimation and submarine melting for a Snowball Earth scenario. That study found that regions with net sublimation also experience a similar magnitude of submarine melt. The special environment of an inland sea may inhibit that process; inland seas are restricted waterways with thick ice that would partially block the underwater channel entrance. However, rift-valley seas can experience enhanced geothermal flux which could thin the ice. Since we do not know whether inland seas in a net-sublimation region would experience net melting or net freezing, we choose to not address that effect in this work.

### 6.3. Atmospheric Constraints

While we can now determine the penetration of a sea glacier into a narrow channel with a narrower entrance for a given mean-annual surface temperature and net sublimation rate, we have not assessed the likelihood of an inland sea to possess those climate conditions. Robust elements of Snowball climate have been determined [Abbot et al., 2013]; however, the climate of an inland sea is a special environment that has not been considered in the previous studies. The inland sea environment is a channel surrounded by land with a sea glacier at the entrance. The only place where a sea glacier cannot penetrate a channel is in a region of net sublimation, which, according to GCM results for Snowball Earth are mostly near the equator [Pierrehumbert et al., 2011; Pollard and Kasting, 2004, Figure 9]. Since inland seas are allowed only in regions of net sublimation, any land areas surrounding the inland sea would likely be desert, covered with bare rock or soil, not snow or ice, although others have indicated that some or all of the land in regions of net sublimation could become covered with land ice flowing from regions of net accumulation [Donnadieu et al., 2003; Pollard and Kasting, 2004; Rodehacke et al., 2013]. If it is unglaciated, the low albedo of the rock and soil surrounding the inland sea would increase absorbed radiation and locally increase the air temperature. If an inland sea is mostly surrounded by bare land, the air temperature and sublimation rate would therefore be greater than on a sea glacier over ocean at the same latitude. The albedo of modern deserts is 0.3–0.4 [Smith, 1986; Otterman and Fraser, 1976] much lower than our estimate for that of a sublimating sea glacier, $\sim 0.6$ [Dadic et al., 2013]. To fully evaluate the likelihood of regional temperatures sufficiently warm to sustain an oasis will require investigation with a general circulation model.

### 6.4. Grounding of the Sea Glacier

An assumption of Campbell et al. [2011] is that the channel is deep enough so the penetrating sea glacier never runs aground (i.e., the sea glacier is always floating). If the Red Sea is again chosen as a suitable analogue, this condition does not hold. The penetrating sea glacier could have an initial thickness as small as 600 m, based on modeling of a water-planet sea-glacier [Goodman, 2006], but the entrance to the Red Sea is only 137 m deep [Siddall et al., 2002]. Clearly, the assumption that the sea glacier is always floating cannot be true everywhere in the channel. This problem can be addressed with by a modification of the ice-shelf equations in Morland [1987] as MacAyeal [1989] has done, or by using a fully 3-D ice-flow model treating grounding-line dynamics.
7. Conclusions

In this work, we tested the ability a sea glacier to penetrate an inland sea with a narrow entrance. We discovered that by reducing the size of the channel entrance, sea-glacier penetration becomes more limited. A sea glacier entering into an inland sea with a warming atmospheric temperature will not penetrate as far into a channel as compared to a channel with uniform atmospheric temperature and sublimation rate. We also discovered that sea-glacier-free zones might exist near the entrances of some channels with narrow entrances, but would not act as refugia at the cold temperatures near the entrance. These results widen the range of climate conditions that allow for inland seas to act as refugia for photosynthetic eukaryotic algae during Snowball Earth events.

Appendix A: Correction to Campbell et al. [2011]

There are two errors in Campbell et al. [2011] that we wish to correct. The cumulative effect of these corrections is that penetrations lengths are approximately 70% greater than reported in our original Figure 2. The first correction was made because of a plotting error in Figure 2 of our paper, hereafter referred to as C2011. This correction made the penetration lengths shown in Figure 2 of C2011 increase by approximately 40%.

The second correction was made because the ice velocities predicted by equation (2) of C2011 are all too small by a factor of 2. This correction caused an increase in penetration length of approximately 20%, affecting both Figures 2 and 3b of C2011. The correction was needed because Nye’s [1965] definition of a temperature-dependent ice-softness $A(T)$ parameter differed by a factor of 2 from the definition of $A(T)$ in current standard usage [e.g., Cuffey and Paterson, 2010, p. 75]. This difference corresponds to a difference of about 6 K in ice temperature $T$. The correct form of equation (2) of C2011 is

$$u(x, y) = W \frac{A(x) k(x)^n}{n+1} \left( 1 - \left| \frac{2y}{W} \right|^n \right).$$

This correction affects some equations from C2011 that follow. The right sides of equations (3), (15), (S-14), (S-15), the two rightmost expressions of equation (S-19), and the left sides of equations (6) and (9) are all too small by a factor of 2, and because of a nonlinear constitutive relationship for ice, equation (14) should have the form

$$D = \left( \frac{2^n b(n+2)}{A T^{2n}} \right)^{\frac{1}{n+1}}.$$

A corrected Figure 2 of C2011 appears here as Figure 6. The penetration length was increased by approximately 70%, further restricting the climatic conditions for which a refugium is allowed. A corrected Figure 3 of C2011 appears here as Figure 7. The bounds for Figure 7b were changed, but the overall shape remains unchanged. [Figure 7a is identical to the published figure.] As modeled here, an analogue to the Red Sea is long enough to have provided a refugium for photosynthetic organisms during Snowball Earth events, only if surface temperature and net sublimation conditions in Figure 6 were met, for example, a $-40$ °C mean-annual surface temperature with a 10 mm/yr net sublimation rate, which seems unlikely.

![Figure 6. Solid contours represent the penetration length-to-width ratio $L/W$ as a function of net sublimation rate $b$ and surface ice temperature $T_s$ for a sea glacier with an initial thickness $H_0 = 650$ m, entering a narrow channel. Dashed line represents the 6.5 $L/W$ ratio for the Red Sea. Atmospheric conditions to the left of the dashed line allow a refugium to avoid being over-ridden at the end of the Red Sea analogue.](image-url)
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References
Nye, J. F. (1965), The flow of a glacier in a channel of rectangular, elliptical or parabolic cross-section, J. Glaciol., 5(41), 661–690.

Figure 7. Top plot (a) shows how increasing atmospheric temperatures during deglaciation affect net sublimation rate $\dot{b}$, relative to an initial value $b_0$ of 1 mm/yr (dashed line), and ice softness $A$, relative to an initial value $A_0$ set by a surface temperature $T_s = -50^\circ$C (solid line). Net sublimation rate $\dot{b}$ is more sensitive than ice softness $A$ to rising surface temperature $T_s$. Bottom plot (b) shows resulting length-to-width ratio $L/W$ of a invading sea glacier, calculated from equation (13). For this particular example, the initial upstream ice thickness is $H_0 = 650$ m.


